Physics 491
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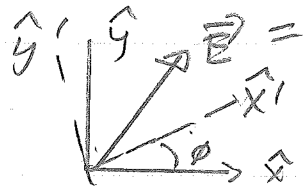
Lecture 11: Photon Spin

Classical E&M wave in vacuum is transverse with $(\vec{E},\vec{B}) \perp \hat{k}$, $\hat{k}$ is direction of propagation. E+M wave carries momentum + angular momentum

Photon has linear polarization states which superimpose to give classical wave. These states transform as the $\vec{E}$ field vector.



$\vec{E} = E_x\hat{x} + E_y\hat{y} = E_x'\hat{x}' + E_y'\hat{y}'$ electric field

Photon state(*)

$|\psi\rangle = \psi_x |\underline{x}\rangle + \psi_y |\underline{y}\rangle$

Euclidean rotation $R^E(+\phi\hat{z}) = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}$

$$\left(|\underline{x}'\rangle, |\underline{y}'\rangle\right) = \left(|\underline{x}\rangle, |\underline{y}\rangle\right)\begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}$$

and ψ_x, ψ_y transform just like E_x, E_y:

$$\begin{pmatrix} \psi_x' \\ \psi_y' \end{pmatrix} = R^E(-\phi\hat{z})\begin{pmatrix} \psi_x \\ \psi_y \end{pmatrix} = \begin{pmatrix} \cos\phi & +\sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix}\begin{pmatrix} \psi_x \\ \psi_y \end{pmatrix}$$

(*) <u>note</u> photon is propagating in $\hat{z}$ direction

Photon angular momentum (spin)

Circular polarization states

$$|\underline{R}\rangle = \frac{1}{\sqrt{2}}\left(|\underline{x}\rangle + i\,|\underline{y}\rangle\right)$$
$$|\underline{L}\rangle = \frac{1}{\sqrt{2}}\left(|\underline{x}\rangle - i\,|\underline{y}\rangle\right)$$

under rotation

$$\hat{R}^E(\phi\hat{z})|\underline{R}\rangle = \frac{1}{\sqrt{2}}\left(\hat{R}^E|\underline{x}\rangle + i\,R^E|\underline{y}\rangle\right)$$
$$= \frac{1}{\sqrt{2}}\left(\cos\phi|\underline{x}\rangle + \sin\phi|\underline{y}\rangle - i\sin\phi|\underline{x}\rangle + i\cos\phi|\underline{y}\rangle\right)$$
$$= (\cos\phi - i\sin\phi)\,\frac{1}{\sqrt{2}}\left(|\underline{x}\rangle + i|\underline{y}\rangle\right) = e^{-i\phi}|\underline{R}\rangle$$

In terms of infinitessimal ϕ, $R^E(\phi\hat{z}) = \left(1 - i\phi\frac{\hat{J}_z}{\hbar}\right)$ giving

$$\frac{\hat{J}_z}{\hbar}|\underline{R}\rangle = +|\underline{R}\rangle$$

similarly $$\frac{\hat{J}_z}{\hbar}|\underline{L}\rangle = -|\underline{L}\rangle$$

states $|\underline{R}\rangle, |\underline{L}\rangle$ are eigenstates of spin with eigenvalues $\pm\hbar$. <u>Photon has spin-1.</u>

$$\left[\frac{\hat{J}_z}{\hbar}\right]_{\text{spherical basis}} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

where $|\underline{R}\rangle \xrightarrow[\text{spherical}]{} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$; $|\underline{L}\rangle \xrightarrow[\text{spherical}]{} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Connection of "spherical" (circular polarization) basis to usual Euclidean rotation.

$$\frac{J_z^E}{\hbar} = i\frac{d}{d\phi}\begin{pmatrix}\cos\phi & -\sin\phi & 0\\ \sin\phi & \cos\phi & 0\\ 0 & 0 & 1\end{pmatrix}\Bigg|_{\phi=0} = i\begin{pmatrix}0 & -1 & 0\\ 1 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}$$

In this basis, none of J_z^E, J_x^E, J_y^E are diagonal

$$\left(|R\rangle, |L\rangle\right) = \left(|\underline{x}\rangle, |\underline{y}\rangle\right)\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ i & -i\end{pmatrix}$$

This change of basis diagonalizes J_z^E.

$$\left[\frac{\hat{J}_z}{\hbar}\right]_{\text{spherical}} = \frac{i}{2}\begin{pmatrix}1 & -i\\ 1 & i\end{pmatrix}\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}\begin{pmatrix}1 & 1\\ i & -i\end{pmatrix}$$

$$= \frac{i}{2}\begin{pmatrix}1 & -i\\ 1 & i\end{pmatrix}\begin{pmatrix}-i & i\\ 1 & 1\end{pmatrix} = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

We could have diagonalized $\hat{J}_z$ and

then transformed from $|\underline{x}\rangle, |\underline{y}\rangle$ to $|R\rangle, |L\rangle$ basis.

From general theory of representation, we will find for spin-1 particle –

$$\left[\frac{J_z}{\hbar}\right]^{\text{spherical}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad \text{(re-ordered)}$$

with states

$$|+1\rangle = \frac{1}{\sqrt{2}}\left(|\underline{x}\rangle + i|\underline{y}\rangle\right)$$

$$|0\rangle = |\underline{z}\rangle$$

$$|-1\rangle = \frac{1}{\sqrt{2}}\left(|\underline{x}\rangle - i|\underline{y}\rangle\right)$$

$|0\rangle$ state (longitudinal polarization) missing for <u>massless</u> photon.

Photon mass (m_γ) would modify Coulomb potential as

$$V(r) = \frac{e^2}{r}\exp\left(-r\frac{m_\gamma c}{\hbar}\right)$$

experimentally, $m_\gamma c^2 < 10^{-18}\,\mathrm{eV}$