Physics 491
M. Gold

Fall 2016

Lecture 16: EPR

1. Entangled state

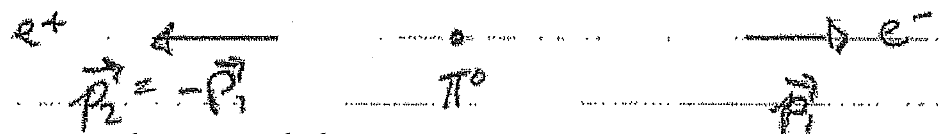
"There is a troubling weirdness about Quantum Mechanics." -S. Weinberg

Multiparticle states that are separable can be written as $|\psi\rangle_1 \otimes |\phi\rangle_2$. States that are not separable are said to be entangled. For example,

$$|0,0\rangle = \frac{1}{\sqrt{2}}\left(|+z\rangle_1 |-z\rangle_2 - |-z\rangle_1 |+z\rangle_2\right)$$

Particles prepared in an entangled state exhibit non-locality.

Example: $\pi^0 \to e^+e^-$ π^0 has $J=0$ so e^+e^- must be in entangled spin state $|0,0\rangle$.



Momentum state is also entangled.

$$\psi = (\text{space}) \otimes |0,0\rangle$$

$\boxed{\text{Quantum non-locality does not conflict with special relativity, as it cannot be used to transfer a signal with } v > c.}$

However, non-locality does conflict with local realism: Measurement of state of particle 1 can not effect state of particle 2 without a physical interaction (which propagates at $v \leq c$).

<u>Quantum Non-locality</u>. The Schrödinger equation is local –

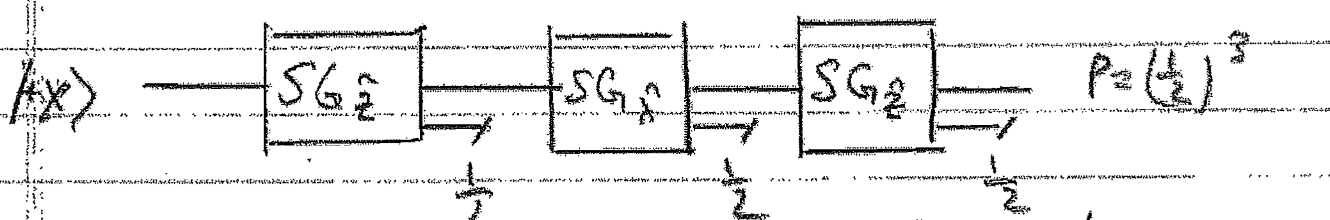
$$i\frac{\partial}{\partial t}\Psi(x,t) = \hat{H}\,\Psi(x,t)$$

does not relate Ψ at different space-time points. Wave function Ψ is <u>non-local</u>.

<u>Hidden Variable theory</u>: Quantum mechanics is incomplete. Local realism can be restored by introducing "hidden" variables. A hidden variable state has values prior to measurement.

To be consistent with non-commuting (incompatible) observables (e.g. $[\hat{S}_x, \hat{S}_y] \neq 0$) must assume that only 1 of incompatible observables is measurable at a time, others remain hidden.

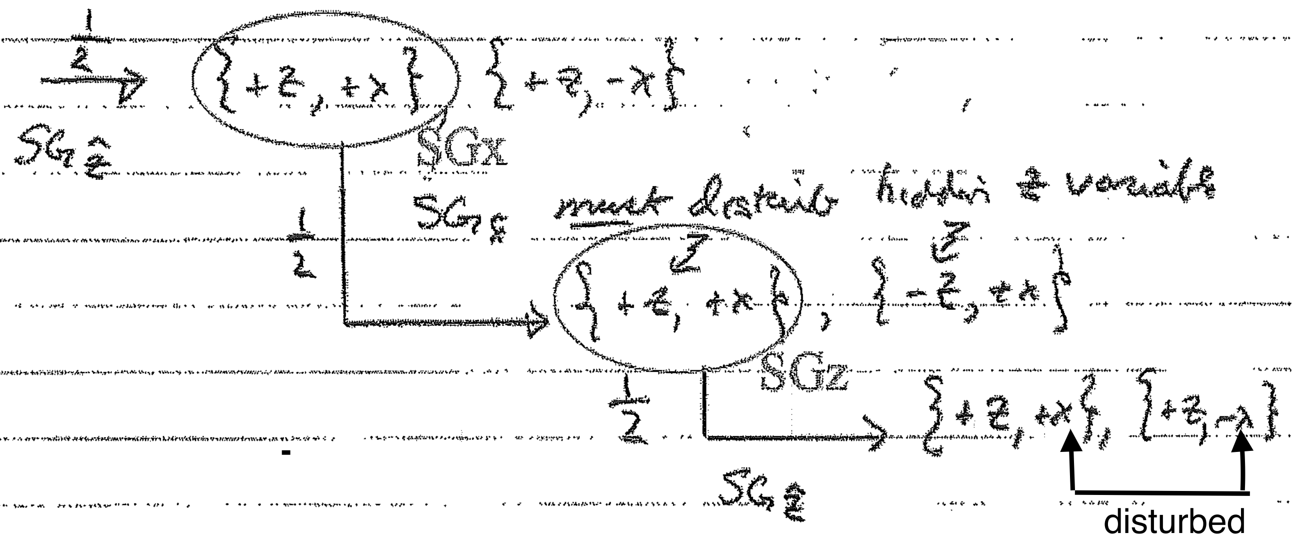
<u>Example</u>: $|+x\rangle = \frac{1}{\sqrt{2}}\left(|+z\rangle + |-z\rangle\right)$



$$P = \left|\langle +z|+x\rangle\langle +x|+z\rangle\langle +z|+x\rangle\right|^2 = \frac{1}{8}$$

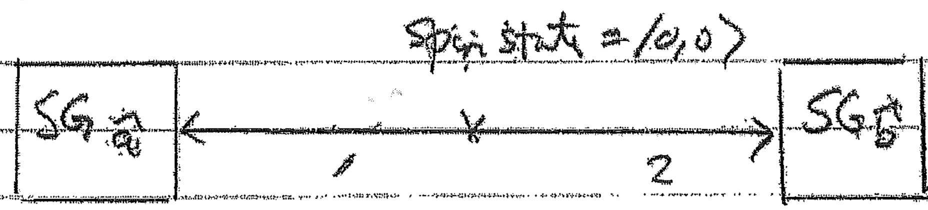
Local realist: 4 possible states, each initially equally likely –

$\{+z, +x\}$, $\{+z, -x\}$, $\{-z, +x\}$, $\{-z, -x\}$



Local realist considers this description preferable to quantum non-locality

"Bell-type" experiment:



$\hat a, \hat b$ are arbitrary directions.

What is $\langle \hat\sigma_b^2 \hat\sigma_a^1 \rangle_{0,0}$?

Write state $|0,0\rangle$ in $\vec{a}$ basis:

$$|0,0\rangle = \frac{1}{\sqrt{2}}\left(|+a\rangle_1 |-a\rangle_2 - |-a\rangle_1 |+a\rangle_2\right)$$

$$\hat{\sigma}_a^{\,1}|0,0\rangle = \frac{1}{\sqrt{2}}\left(|+a\rangle_1 |-a\rangle_2 + |-a\rangle_1 |+a\rangle_2\right)$$

$$\langle 0,0|\hat{\sigma}_b^{\,2}\hat{\sigma}_a^{\,1}|0,0\rangle =$$

$$\frac{1}{2}\left({}_1\langle +a|\,{}_2\langle -a| - {}_1\langle -a|\,{}_2\langle +a|\right)\hat{\sigma}_b^{\,2}\left(|+a\rangle_1|-a\rangle_2 + |-a\rangle_1|+a\rangle_2\right)$$

$$= \frac{1}{2}\left({}_2\langle -a|\hat{\sigma}_b^{\,2}|-a\rangle_2 - {}_2\langle +a|\hat{\sigma}_b^{\,2}|+a\rangle_2\right)$$

<u>where we used</u> ${}_1\langle \pm z|\mp z\rangle_1 = 0$, ${}_1\langle \pm z|\pm z\rangle_1 = 1$

without loss of generality, take $\hat{a} = \hat{z}$
and $\hat{b} = \cos\theta\,\hat{z} + \sin\theta\,\hat{x}$

$$\hat{\sigma}_b = \begin{pmatrix} c & s \\ s & -c \end{pmatrix}$$

$$(0,1)\begin{pmatrix} c & s \\ s & -c \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = -c$$

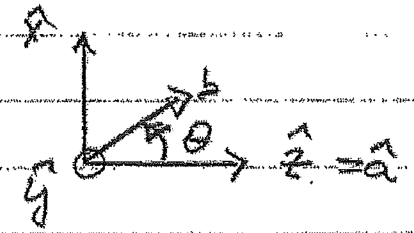
$$(1,0)\begin{pmatrix} c & s \\ s & -c \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = +c$$

$$\boxed{\langle 0,0|\hat{\sigma}_b^{\,2}\hat{\sigma}_a^{\,1}|0,0\rangle = -\cos\theta = -\hat{a}\cdot\hat{b}}$$

Probabilities in Bell-type experiment

4 possible outcomes for particle (1,2) –
(+a, +b) (+a, −b) (−a, +b) (−a, −b)

Without loss of generality:



$$\left(|+b\rangle, |-b\rangle\right) = \left(|+a\rangle, |-a\rangle\right)\begin{pmatrix} c & -s \\ s & c \end{pmatrix}$$

where $c = \cos\theta/2$, $s = \sin\theta/2$

$$|+b\rangle = \cos\tfrac{\theta}{2}\,|+a\rangle + \sin\theta/2\,|-a\rangle$$
$$|-b\rangle = -\sin\tfrac{\theta}{2}\,|+a\rangle + \cos\tfrac{\theta}{2}\,|-a\rangle$$

initial state

$$|0,0\rangle = \frac{1}{\sqrt{2}}\left(|+a,-a\rangle - |-a,+a\rangle\right)$$

$$A(+a,+b) = \langle +a, +b|0,0\rangle = \frac{1}{\sqrt{2}}\;{}_2\langle +b|-a\rangle_2$$ $=\langle -a|+b\rangle^* = (1/\sqrt{2})\sin(\theta/2)$

$$A(+a,-b) = \langle +a,-b|0,0\rangle = \frac{1}{\sqrt{2}}\;{}_2\langle -b|-a\rangle_2$$

etc.

outcome 1, 2	Amplitude ${}_2\langle\ \mid\ \rangle_2$	P(1,2)
+a, +b	$1/\sqrt{2}\ \langle +b\|-a\rangle$	$\frac{1}{2}\sin^2\theta/2$
+a, −b	$1/\sqrt{2}\ \langle -b\|-a\rangle$	$\frac{1}{2}\cos^2\theta/2$
−a, +b	$-1/\sqrt{2}\ \langle +b\|+a\rangle$	$\frac{1}{2}\cos^2\theta/2$
−a, −b	$+1/\sqrt{2}\ \langle -b\|+a\rangle$	$\frac{1}{2}\sin^2\theta/2$

↖ wave function factor

note $\sum P_i = 1$

$$\langle \hat{\sigma}_b^2\, \hat{\sigma}_a^1\rangle = P(+a,+b) + P(-a,-b) - P(+a,-b) - P(-a,+b)$$
$$= \sin^2\theta/2 - \cos^2\theta/2 = -\cos\theta \quad \text{as } \underline{\text{before}}$$

hidden variable analysis:

Outcome	hidden variable state
$+a, +b$	$\{+a,-b\}_1 \ \{-a,+b\}_2$
$+a, -b$	$\{+a,+b\}_1 \ \{-a,-b\}_2$
$-a, +b$	$\{-a,-b\}_1 \ \{+a,+b\}_2$
$-a, -b$	$\{-a,+b\}_1 \ \{+a,-b\}_2$

Somehow, the probabilities for each arrangement in the ensemble must be set in advance to correspond to Q.M. result. Very implausible to me!

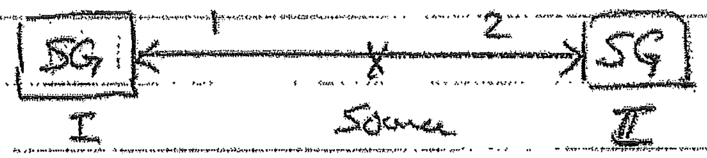
Classical correlations are set in advance.

Example Bell Inequality:

Consider again entangled spin state

$$|0,0\rangle = \frac{1}{\sqrt{2}}\left(|+z\rangle_1 |-z\rangle_2 - |-z\rangle_1 |+z\rangle_2 \right)$$

in experiment with 2 SG measurements.



SG I, II can measure each of $\hat{a}, \hat{b}, \hat{c}$ spin directions.

Hidden variable theory requires each particle "state" to have values for $\hat{a}, \hat{b}, \hat{c}$ outcome of $\pm \frac{\hbar}{2}$; 2^3 arrangements.

100% anti-correlated

	Number	particle 1	particle 2	
	N_1	$+a$ $+b$ $+c$	$-a$ $-b$ $-c$	
	N_2	$+a$ $+b$ $-c$	$-a$ $-b$ $+c$	$P(+a_1,+c_2)$
$P(+a_1,+b_2)$	N_3	$+a$ $-b$ $+c$	$-a$ $+b$ $-c$	
	N_4	$+a$ $-b$ $-c$	$-a$ $+b$ $+c$	
	N_5	$-a$ $+b$ $+c$	$+a$ $-b$ $-c$	
	N_6	$-a$ $+b$ $-c$	$+a$ $-b$ $+c$	
	N_7	$-a$ $-b$ $+c$	$+a$ $+b$ $-c$	$P(+c_1,+b_2)$
	N_8	$-a$ $-b$ $-c$	$+a$ $+b$ $+c$	

where N_i could be chosen to fit experiment.

However, we must always have

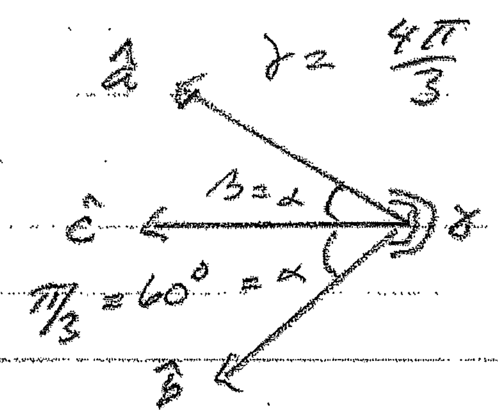
$$P(+a_1, +b_2) \leq P(+a_1, +c_2) + P(+c_1, +b_2)$$

Hence $N_3 + N_4 \leq (N_2 + N_4) + (N_3 + N_7)$

Q.M. prediction is

$$\frac{1}{2}\sin^2\left(\frac{\gamma}{2}\right) \overset{?}{\leq} \frac{1}{2}\sin^2\left(\frac{\beta}{2}\right) + \frac{1}{2}\sin^2\left(\frac{\alpha}{2}\right)$$

which will violate inequality. For example, take $\gamma = 2\pi - \alpha - \beta$ (3 directions in a plane) and $\alpha = \beta = \pi/3$



$$\sin^2(120^\circ) \overset{?}{\leq} 2\sin^2(30^\circ)$$

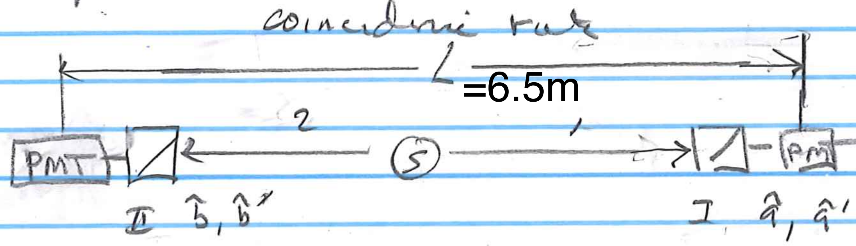
$$\left(\frac{\sqrt{3}}{2}\right)^2 \overset{?}{\leq} 2\left(\frac{1}{2}\right)^2$$

$$\frac{3}{4} > \frac{1}{2}$$

Q.M. violates Bell's inequality

Experimental test of Bell Inequality

Aspect et al. PRL 47 (460) 1981
Aspect et al. PRL 49 (1804) 1982



L/c = 21.7 ns

linear polarizers I, II

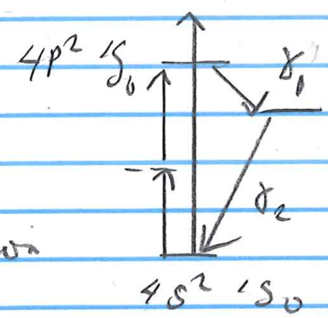
measure PMT coincidence rates: $\hat{a}, \hat{b}$; $\hat{a}, \hat{b}'$
$\hat{a}', \hat{b}$; $\hat{a}', \hat{b}'$

switch synchronously between 4 possibilities

Δt=5ns < L/c

<u>Source: Ca</u>

laser
2γ coherent
absorption



$J=0, L=0$

$J=1$ L=1

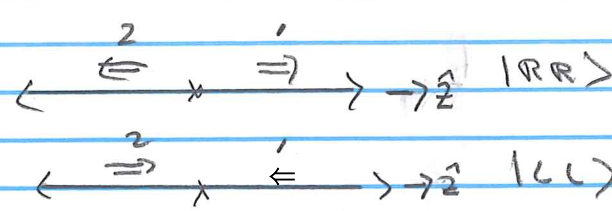
$J=0, L=0$

notation ${}^{2S+1}L_J$ in transitions electron <u>spin</u> does not change

Two photon state has total $J=0$

$$|0,0\rangle = \frac{1}{\sqrt{2}}\left(|RR\rangle + |LL\rangle\right)$$

+ sign not obvious see Hw 5.13



photon helicity states with $J=0$

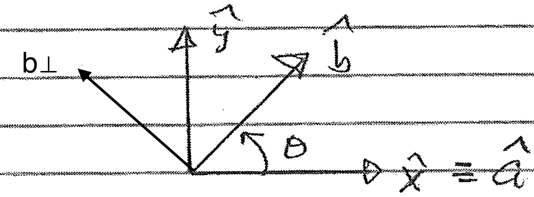
Be careful about direction of rotation of classical $\vec{E}$ vector polarization

$$|R\rangle_1 = (|x\rangle_1 + i|y\rangle_1)\frac{1}{\sqrt{2}} = |L\rangle_2$$

$$|L\rangle_1 = (|x\rangle_1 - i|y\rangle_1)\frac{1}{\sqrt{2}} = |R\rangle_2$$

See HW 5.13

$$|00\rangle = \frac{1}{\sqrt{2}}\left(|x\rangle_1|x\rangle_2 + |y\rangle_1|y\rangle_2\right)$$



$|x\rangle, |y\rangle$ basis refer to polarization along $\hat{x}, \hat{y}$ direction

$|b\rangle$ polarization is linear combination or left handed rotation of components from $\hat{x}$ to $\hat{b}$: $|x\rangle \to \begin{pmatrix}1\\0\end{pmatrix}$ $|y\rangle \Rightarrow \begin{pmatrix}0\\1\end{pmatrix}$

$|b\rangle = b_1|x\rangle + b_2|y\rangle$

$$\begin{pmatrix}b_1\\b_2\end{pmatrix} = \begin{pmatrix}c & -s\\ +s & c\end{pmatrix}_\theta \begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}\cos\theta\\ +\sin\theta\end{pmatrix}$$

orthogonal state polarized $\perp$ to $\hat{b}$ is:

$|b_\perp\rangle = -\sin\theta\,|x\rangle + \cos\theta\,|y\rangle$

$|b_\parallel\rangle = \cos\theta\,|x\rangle + \sin\theta\,|y\rangle$

polarization $\perp$, along $\hat{b}$ direction

Expect $\langle b, x | 0,0 \rangle = \frac{1}{\sqrt{2}} {}_2\langle b | x \rangle_2 = \frac{1}{\sqrt{2}} \cos\theta$

$x = a$

$|\langle b, a | 0,0 \rangle|^2 = \frac{1}{2} \cos^2\theta$ normalized coincidence rate

measured coincidence rate agree with this (PRL 1981)

Bell inequality is generalized "CHSH" Bell inequality

$R(\hat{a}, \hat{b})$ normalized coincidence rate

$$S \equiv R(\hat{a}, \hat{b}) - R(\hat{a}, \hat{b}') + R(\hat{a}', \hat{b}) + R(\hat{a}', \hat{b}') - R(\hat{a}', -) - R(-, \hat{b})$$

where "$-$" means corresponding polarizer removed

Bell inequality

$$-1 \leq S \leq 0$$

$S_{exp} = 0.126 \pm 0.014$ PRL 1981

$S_{\substack{QM \\ theory}} = 0.118 \pm 0.005$

$-2 \leq S' \leq 2$ $S_{exp} = 2.697 \pm 0.015$ PRL 1982

different Bell S' $S_{\substack{qm \\ theory}} = 2.70 \pm 0.05$

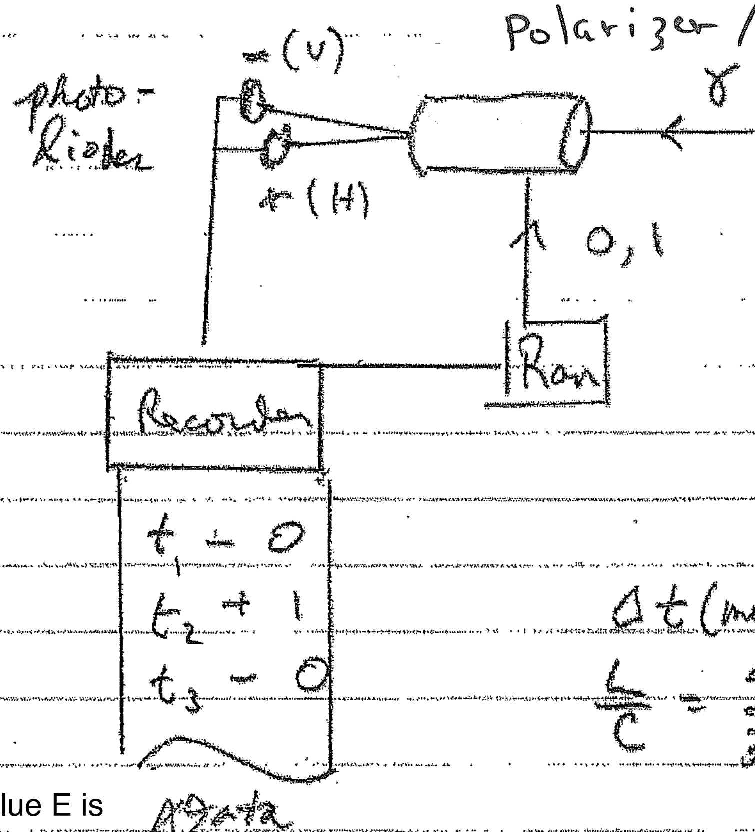
Weihs et al., PRL 81 (5039) 1998

Loopholes ① inefficient detectors
② spacelike separation of "observers" (not sinusoidal switching like Aspect)

Source – "degenerate type-II parametric down conversion"

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|H\,V\rangle - |V\,H\rangle\right)$$

observers (Alice, Bob)
~~simplified~~ modulated



1 of 2 observers

0: θ
1: θ+45°
θ = α, β
for 2 observers

$\Delta t(\text{measure}) < 0.1\,\mu s$

$$\frac{L}{C} = \frac{400\,m}{300\,m/\mu s} = 1.3\,\mu s$$

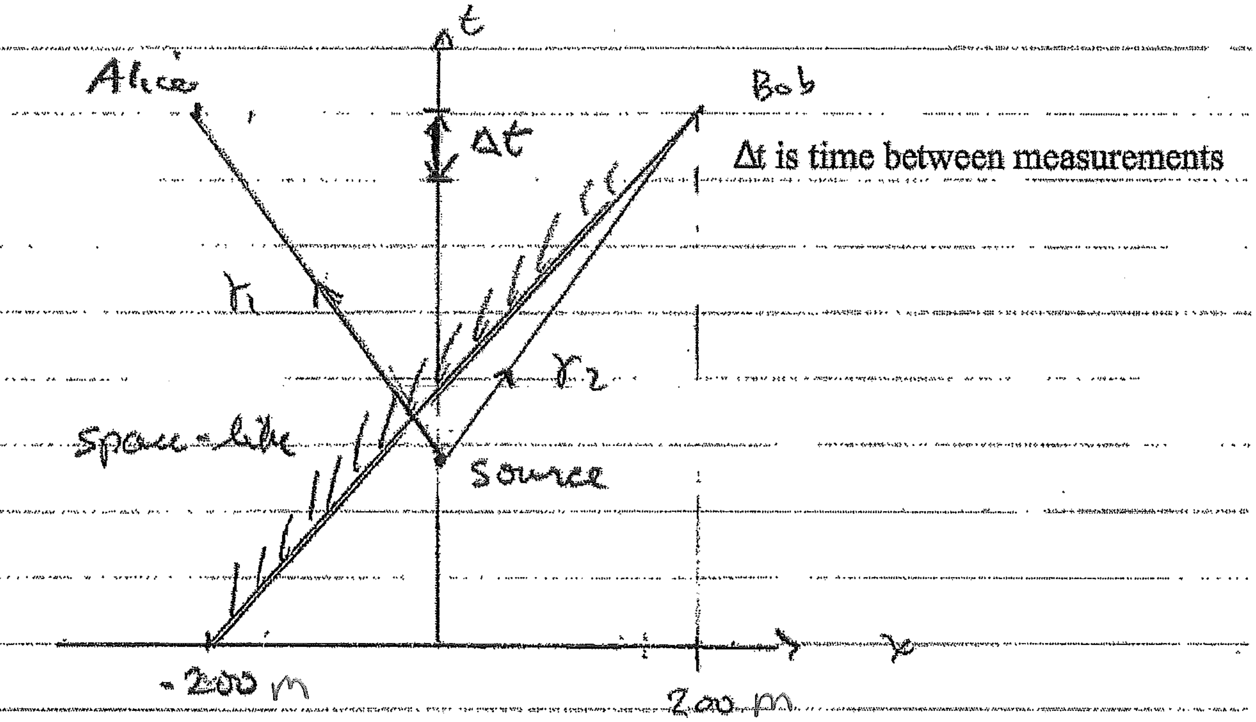
Expectation value E is

$$E(\alpha,\beta) \equiv \frac{1}{N}\left(C_{++} + C_{--} - C_{+-} - C_{-+}\right) \quad C \equiv \text{COUNT}$$

$C_{++}^{QM} \propto \sin^2(\beta-\alpha)$ QM predicted counts

$$E^{QM}(\alpha,\beta) = -\cos(2(\beta-\alpha))$$

space-time diagram



light fiber 250 m length

Generalized Bell inequality –

$$S(\alpha,\alpha',\beta,\beta') = \left|E(\alpha,\beta) - E(\alpha',\beta)\right| + \left|E(\alpha,\beta') + E(\alpha',\beta')\right| \leq 2$$

$$S^{QM}_{max}(0, 45^\circ, 22.5^\circ, 67.5^\circ) = 2\sqrt{2} = 2.82$$

due to imperfect correlation "visibility" of source (97%) expect $S \simeq 2.74$

$$S^{exp.} = 2.73 \pm 0.02$$

"Expecting that any improved experiment will also agree with quantum theory, a shift of our classical philosophical positions seems necessary." Hensen et al.

$$N = 14700$$

"Loophole-free Bell inequality violation using electron spins separated by 1.3 kilometres", Hensen et al., 682 I NATURE I VOL 526 I 29 OCTOBER 2015