Physics 491 Fall 2015

Recitation 1: Solutions

(1) $$\langle E\rangle = \sum E\,P(E) = \frac{\sum n\varepsilon e^{-n\varepsilon/kT}}{\sum e^{-n\varepsilon/kT}}$$

let $x = \varepsilon/kT$, $$\langle E\rangle = kT\,\frac{\sum nx\,e^{-nx}}{\sum e^{-nx}} = kT\,\frac{-x\frac{\partial}{\partial x}\sum e^{-nx}}{\sum e^{-nx}}$$

$$S(x) = \sum e^{-nx} = 1 + e^{-x} + e^{-2x} + \cdots = 1 + e^{-x} S(x)$$

$$S(x) = \frac{1}{1-e^{-x}} \;;\quad \frac{dS}{dx} = \frac{-e^{-x}}{(1-e^{-x})^2}$$

$$\langle E\rangle = \frac{\varepsilon e^{-\varepsilon/kT}}{1-e^{-\varepsilon/kT}} = \frac{\varepsilon}{e^{\varepsilon/kT}-1}$$

$\varepsilon = hf$ gives Planck law

(2) $$m + P_\gamma = P_\gamma' + E$$
$$0 = P_{ey} + P_\gamma' \sin\theta$$
$$P_\gamma = P_{ex} + P_\gamma' \cos\theta$$

$$E_e^2 = m^2 + (P_\gamma - P_\gamma'\cos)^2 + (P_\gamma'\sin)^2 = m^2 + P_\gamma^2 + P_\gamma'^2 - 2P_\gamma P_\gamma'\cos$$

$$= (m + P_\gamma - P_\gamma')^2 = m^2 + 2m(P_\gamma - P_\gamma') + (P_\gamma - P_\gamma')^2$$

$$P_\gamma P_\gamma'(1-\cos) = m(P_\gamma - P_\gamma')$$

$$\left(\frac{1}{P_\gamma'} - \frac{1}{P_\gamma}\right) = \frac{1}{m}(1-\cos) \quad \text{or} \quad \boxed{\lambda' - \lambda = \frac{h}{m}(1-\cos)}$$