Physics 491: Recitation #1
August 30, 2022

1. Planck's radiation law for the energy density in the black body cavity as a function of wavelength $u(\lambda, T)$, by a change of variables to frequency $f = c/\lambda$, is $u(f,t) = (c/f^2)u(\lambda, T)$. The energy density as a function of frequency has interpretation as the product of the number of modes per unit energy times the mean energy per mode of the EM field
$$u(f,t) = \frac{8\pi f^2}{c^3}\langle E\rangle = \frac{8\pi f^2}{c^3}\left(\frac{hf}{e^{hf/kT}-1}\right)$$
To derive his law, Planck assumed that $\langle E\rangle$ was calculated with the assumption that energy is quantized as $E = n\epsilon$ and the Boltzmann probability
$$P(E) = \frac{e^{-n\epsilon/kT}}{\sum\limits_{n=0}^{\infty} e^{-n\epsilon/kT}}$$
.

   Calculate $\langle E\rangle$ and show $\epsilon = hf$.

   Hints: By rearranging the sum in the denominator (call it $S(x)$ where $x = \epsilon/kT$) you can find an equation for $S(x)$ to solve for it. Then the numerator in the average can be written in terms of the derivative of S.

2. Derive the Compton scattering formula using relativistic energy and momentum with photon energy $E = hf = hc/\lambda$. It is convenient to choose units so that the speed of light $c = 1$. Then with the scattering in the x-y plane and writing 4-momenta as $(E, p_x, p_y, 0)$ (I will not write the p_z in the following). We have the 4-momentum as:
$$\begin{aligned} &\text{initial photon} \quad (p, p, 0) \\ &\text{initial electron} \quad (m, 0, 0) \end{aligned}$$
$$\begin{aligned} &\text{final photon} \quad (p', p'\cos\theta, p'\sin\theta) \\ &\text{final electron} \quad (E_e, p_{ex}, p_{ey}) \end{aligned}$$
You find
$$\lambda' - \lambda = \frac{h}{m}(1-\cos\theta)$$
(you can now put in the factor of c to get the units correct)