Physics 491: Recitation #11
November 18, 2016

1. Consider a system of two spin-1/2 particles. What are the possible eigenvalues of the operator $\hat{\vec{S}}_1 \cdot \hat{\vec{S}}_2$ on eigenstates of the total spin operator? Comment on any degeneracies.

2. Consider a state of three spin-1/2 particles. Determine the states of total spin quantum number $s = 3/2$ by applying the lowering operator. It is convenient to use the notation
$$|1/2, +1/2\rangle_1 \, |1/2, +1/2\rangle_2 \, |1/2, +1/2\rangle_3 = |+++\rangle \ \text{, etc.}$$
There are a total of four spin $s = 3/2$ states. From combining three spin-1/2 particles you can get $2 \times 2 \times 2 = 8$ combinations. There are in addition to the four spin $s = 3/2$ states, two spin $s = 1/2$ doublets that are orthogonal mixtures of the three spinor states.

3. In this problem we apply rotational symmetry to a general, Hermitian, scalar operator that depends linearly on the spin-1/2 operator $\hat{\vec{S}}$
$$\hat{A} = a + 2\vec{b} \cdot \left(\frac{\hat{\vec{S}}}{\hbar} \right)$$
where a is a scalar $\vec{b}$ is a vector. Consider an arbitrary function $f(\hat{A})$ defined by its Taylor series expansion about zero,
$$f(\hat{A}) = f(0)\hat{I} + f'(0)\hat{A} + \frac{1}{2!} f''(0)\hat{A}^2 + \dots$$
Then it must be possible to write $f(\hat{A})$ in the same general linear form,
$$f(\hat{A}) = \alpha + 2\beta\vec{b} \cdot \left(\frac{\hat{\vec{S}}}{b\hbar} \right)$$
Consider the eigenvalues of $\hat{A}$ and $f(\hat{A})$ and relate the constants α, β to a, b.