Physics 491

Recitation #2 Solutions

(1) A harmonic oscillator has 2 degrees of freedom. The equipartition theorem gives the energy of a 1D oscillator as $2\left(\frac{1}{2}kT\right) = kT$.

$$C_V = \frac{\partial U}{\partial T} = \frac{\partial}{\partial T}(3RT) = 3R \qquad (R = N_0 k)$$

With $\langle E \rangle = \dfrac{hf}{e^{hf/kT}-1}$ the mean 1D oscillator energy,

oscillator

$$U = \frac{3N_0\, hf}{e^{hf/kT}-1} \xrightarrow[T\to\infty]{} 3RT \quad \text{and} \quad C_V \xrightarrow[T\to\infty]{} 3R$$

(2) $$\langle p_x \rangle = \int_{-a/2}^{a/2} \phi_n^*(x)\, \frac{\hbar}{i}\frac{\partial}{\partial x}\, \phi_n(x)\, dx$$

with $\phi_n(x) = \begin{cases} \sin k_n x \\ \cos k_n x \end{cases}$ $\qquad \displaystyle\int_{-a/2}^{a/2} \sin kx \cos kx = 0$

so $\langle p_x \rangle = 0$.

(3) $$\langle p_x \rangle = \int \psi^* \frac{\hbar}{i}\frac{\partial}{\partial x}\psi\, dx$$

$$\langle p_x \rangle^* = \int \psi \left(-\frac{\hbar}{i}\frac{\partial}{\partial x}\right) \psi^* dx \underset{\uparrow}{=} \langle p_x \rangle$$

integrate by parts