Physics 491: Recitation #2
August 31, 2022

1. Einstein considered the thermal vibrations of a solid to be equivalent to a large number of 3 dimensional oscillators all at the same frequency f.

   In classical statistical mechanics, the equipartition theorem says each degree of freedom has average energy $kT/2$ where T is the absolute temperature and k is Boltzmann's constant. Show that the heat capacity per mole at constant volume, $C_V = \partial U/\partial T = 3R$, where U is the energy per mole of the solid. (Hint: How many degrees of freedom does a harmonic oscillator have?)

   Now use the average oscillator energy $\langle E \rangle$ from quantum statistics to get C_V as a function of T, and show that it reduces to $3R$ at high temperature. From recitation 1, recall that the mean energy per harmonic oscillator mode with quantized energy ϵ is
$$\langle E \rangle = \frac{\epsilon}{e^{\epsilon/kT} - 1}$$

2. What is the $\langle p_x \rangle$ for a particle in a box in an arbitrary state?

3. The expectation of the momentum must be real (a physical value). Yet, the corresponding operator $\hat{p_x} = (\hbar/i)\partial/\partial x$ "looks complex" . Prove that the expectation value is always real. Such and operator with real eigenvalues is called Hermitian.