

Recitation 3 Solutions

①  $E = \frac{p^2}{2m} - \frac{\hbar c \alpha}{r}$  estimate  $p \sim \frac{\hbar}{r}$   
a lucky guess

$$E(r) = \frac{\hbar^2}{2mr^2} - \frac{\hbar c \alpha}{r}$$

$$\frac{dE}{dr} \Big|_{r_{\min}} = 0 = \frac{-\hbar^2}{2mr_{\min}^3} + \frac{\hbar c \alpha}{r_{\min}^2}$$

$$r_{\min} = \frac{\hbar}{m c \alpha} = a_0$$

$$E(r_{\min}) = -\frac{1}{2} m c^2 \alpha^2$$

②  $E = \frac{p^2}{2m} + mgz$  estimate  $p \sim \frac{\hbar}{z}$

$$E(z) = \frac{\hbar^2}{2m} \left( \frac{1}{z^2} \right) + mgz$$

$$\frac{dE}{dz} \Big|_{z_{\min}} = 0 \quad \text{guess} \quad z_{\min}^3 = \frac{\hbar^2}{m^2 g} = \frac{(\hbar c)^2}{m c^2 (m g)}$$

$$z_{\min}^3 = \frac{(0.2 \text{ eV} \cdot \mu\text{m})^2}{10^9 \text{ eV} \times 10^{-13} \text{ eV}/\mu\text{m}} = 400 \mu\text{m}^3$$

$$z_{\min} \approx 7 \mu\text{m}$$