Physics 491
9/18/15

Recitation #5 Solutions

① since $\mu \propto \frac{1}{mass}$ and $m_e/m_p \sim \frac{1}{2000}$

$\mu_N = 3\times10^{-8}\,eV/T$

g factors $\neq 2$ means they are composite particles

② Most general 2x2 Hermetian matrix

$$\sigma_x = \begin{pmatrix} a & b \\ b^* & c \end{pmatrix} \qquad a, c \text{ real}$$

$$\sigma_x \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ \pm1 \end{pmatrix} = \pm\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ \pm1 \end{pmatrix}$$

$$\begin{pmatrix} a \pm b \\ b^* \pm c \end{pmatrix} = \pm\begin{pmatrix} 1 \\ \pm1 \end{pmatrix}$$

$a+b=1$ ①
$a-b=-1$ ②
$b^*+c=+1$ ③
$b^*-c=+1$ ④

from ①+②, $a=0$
from ③+④, $b^*=1=b$ (real)
④−③ $\Rightarrow c=0$

so $\boxed{\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}}$

③ $\langle \hat{S}_z \rangle = \frac{\hbar}{2}$ since it is a vector, $\langle \hat{S}_{z'} \rangle = \frac{\hbar}{2}\cos\theta$

with $|+z\rangle = \alpha\,|+z'\rangle + \beta\,|-z'\rangle$

$$\langle S_z' \rangle = \frac{\hbar}{2}\cos\theta = \frac{\hbar}{2}\alpha^2 - \frac{\hbar}{2}\beta^2$$

also $\langle +z|+z\rangle = 1 = \alpha^2+\beta^2$

so
$$\alpha^2 = \frac{1}{2}(1+\cos\theta) = \cos^2\frac{\theta}{2}$$
$$\beta^2 = \frac{1}{2}(1-\cos\theta) = \sin^2\frac{\theta}{2}$$