Physics 491 Fall 2016

Recitation #8 Solutions

1) $|\psi\rangle \xrightarrow{z} N\begin{pmatrix}1\\2\\3i\end{pmatrix}$ $N = \dfrac{1}{\sqrt{1^2+2^2+3^2}} = \dfrac{1}{\sqrt{14}}$

$P_{+1} = \dfrac{1}{14}$; $P_0 = \dfrac{4}{14}$; $P_{-1} = \dfrac{9}{14}$

$\hat{S}_z = \hbar \, \mathrm{diag}(1, 0, -1)$

$$\langle S_z \rangle = \frac{\hbar}{14}(1, 2, -3i)\begin{pmatrix}1&0&0\\0&0&0\\0&0&-1\end{pmatrix}\begin{pmatrix}1\\2\\3i\end{pmatrix} = -\frac{8\hbar}{14} = -\frac{4}{7}\hbar$$

$$\langle S_x \rangle = \frac{\hbar}{\sqrt{2}(14)}(1, 2, -3i)\underbrace{\begin{pmatrix}0&1&0\\1&0&1\\0&1&0\end{pmatrix}\begin{pmatrix}1\\2\\3i\end{pmatrix}}_{\begin{pmatrix}2\\1+3i\\2\end{pmatrix}}$$

$$= \frac{\hbar}{14\sqrt{2}}\left[2 + 2(1+\cancel{3i}) - \cancel{3i}(2)\right] = +\frac{2}{7\sqrt{2}}\hbar$$

#2 $\hat{H} = \omega_0 \hat{S}_x \rightarrow \left[\frac{\omega_0 \hbar}{2} \sigma_x\right]_{z \text{ basis}}$

$$\left[e^{-i\hat{H}t/\hbar}\right]_z = I - i\frac{\omega_0 t}{2}\sigma_x - \left(\frac{\omega_0 t}{2}\right)^2 \frac{\sigma_x^2}{2!} - i\left(\frac{\omega_0 t}{2}\right)^3 \frac{\sigma_x^3}{3!}$$

with $\sigma_x^2 = I$, $= \underline{I}\cos\frac{\omega_0 t}{2} - i\sigma_x \sin\frac{\omega_0 t}{2}$

$$= \begin{pmatrix} \cos\frac{\omega_0 t}{2} & -i\sin\omega_0 t/2 \\ -i\sin\frac{\omega_0 t}{2} & -\cos\frac{\omega_0 t}{2} \end{pmatrix}$$

$$e^{-i\hat{H}t/\hbar}|+z\rangle \xrightarrow[z]{} \begin{pmatrix} \cos\frac{\omega_0 t}{2} \\ -i\sin\frac{\omega_0 t}{2} \end{pmatrix}$$

$$\langle \hat{S}_z \rangle = \frac{\hbar}{2}\left(c, -i^* s\right)\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} c \\ is \end{pmatrix}$$

$$= \frac{\hbar}{2}\left(c^2 - s^2\right) = \frac{\hbar}{2}\cos(\omega_0 t)$$