Physics 491
Fall 2015

Recitation #9 Solutions

(4)
$$\omega = 2\pi f = g\,\mu_e\left(\frac{m_e}{m_\mu}\right)\frac{B}{\hbar}$$

$$g = \frac{h f}{\mu_e B}\left(\frac{m_\mu}{m_e}\right)$$

$h = 4.1\times10^{-21}$ MeV·s

$\mu_e = \left(\frac{e\hbar}{2m_e c}\right) = 5.8\times10^{-11}$ MeV/T

$\frac{m_\mu}{m_e} = 210$

$B = 60\,G = 6\times10^{-3}\,T$

$f = \frac{1}{1.24\times10^{-6}\,s}$ from reading graph

$$g \cong \frac{4.1\times10^{-21}\,\text{MeV·s}\left(\frac{1}{1.3\times10^{-6}\,s}\right)210}{5.8\times10^{-11}\,\text{MeV/T}\,(6\times10^{-3}\,T)}$$

$$\cong \frac{4.1\times210}{5.8\times(1.24)6}\;10^{-21+\underbrace{6+11}_{-1}+3} = 2$$

(#2) $\hat{H} = -\omega_0 \hat{S}_z \qquad \omega_0 = \frac{gqB}{2mc} \qquad q>0$

$$|\psi(0)\rangle = \tfrac{1}{2}|1,1\rangle + \frac{i\sqrt{2}}{2}|1,0\rangle - \tfrac{1}{2}|1,-1\rangle$$

$$\underset{z}{\longrightarrow} \frac{1}{2}\begin{pmatrix} 1 \\ i\sqrt{2} \\ -1 \end{pmatrix}$$

$$|\psi(t)\rangle \underset{z}{\longrightarrow} \frac{1}{2}\begin{pmatrix} e^{i\omega_0 t} \\ i\sqrt{2} \\ -e^{-i\omega_0 t} \end{pmatrix}$$

$$\hat{S}_z \longrightarrow \hbar\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\langle \hat{S}_z \rangle = \frac{\hbar}{4}\left(e^{-i\omega_0 t}, -i\sqrt{2}, -e^{+i\omega_0 t}\right)\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}\begin{pmatrix} e^{i\omega_0 t} \\ i\sqrt{2} \\ -e^{-i\omega_0 t} \end{pmatrix}$$

$$= 0$$

$$\hat{S}_x \longrightarrow \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\langle \hat{S}_x \rangle = \frac{\hbar}{\sqrt{2}}\left(\frac{1}{4}\right)\left(e^{-i\omega_0 t}, -i\sqrt{2}, -e^{i\omega_0 t}\right)\underbrace{\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} e^{i\omega_0 t} \\ i\sqrt{2} \\ -e^{-i\omega_0 t} \end{pmatrix}}$$

$$2i \sin\omega_0 t \rightarrow \begin{pmatrix} i\sqrt{2} \\ e^{i\omega_0 t} - e^{-i\omega_0 t} \\ i\sqrt{2} \end{pmatrix}$$

$$= \frac{\hbar}{2}\sin\omega_0 t$$

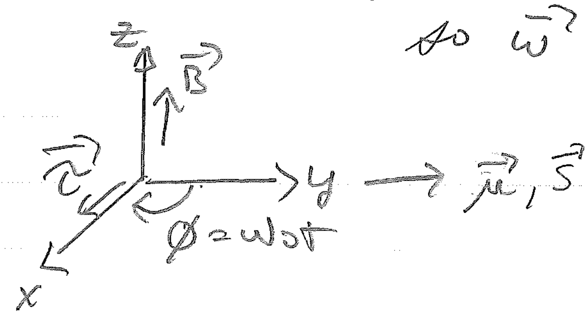
corresponding to precession about $\hat{z}$ ... (next page)

Check the direction of classical precession,

$$\vec{\tau} = \vec{\mu} \times \vec{B} = \frac{d\vec{S}}{dt} = \vec{\omega} \times \vec{S}$$

at $t=0$, $\vec{\mu} \times \vec{B} = \omega_0 \hat{y} \times \hat{z} = \omega_0 \hbar \hat{x}$

so $\vec{\omega} = -\omega_0 \hat{z}$



Consistent with $\langle \hat{S}_x \rangle = +\frac{\hbar}{2} \sin \omega_0 t$