Phys 491
Fall 2022

Example 4.11 Spin-1 Particle

Spin 1 particle charge q mass m in uniform $\vec{B}$ field.
At $t=0$, particle is in an eigenstate of $\hat{S}_y$.
Find $\langle S_x\rangle, \langle S_y\rangle, \langle S_z\rangle$ at time t. $\hat{H} = -\vec{\mu}\cdot\vec{B} = -\left(\frac{qgB}{2mc}\right)\hat{S}_z = -\omega_0\hat{S}_z$

from $[\hat{S}_y]^2 = \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}$ solve eigenvalue $[S_y](\ ) = \mu\hbar(\ )$

see p. 87

$$|\psi(0)\rangle = |1,1\rangle_y \xrightarrow[z]{} \frac{1}{2}\begin{pmatrix} 1 \\ i\sqrt{2} \\ -1 \end{pmatrix}$$

$$|\psi(t)\rangle \xrightarrow[z]{} \frac{1}{2}\begin{pmatrix} e^{i\omega_0 t} \\ i\sqrt{2} \\ -e^{-i\omega_0 t} \end{pmatrix}$$

$$\langle S_z\rangle = \langle\psi(t)|\hat{S}_z|\psi(t)\rangle = (\quad)^* \hbar\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}(\quad) = 0$$

$$\langle S_y\rangle = \frac{1}{4}(\quad)^* \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}(\quad)$$

$$= \frac{1}{4}\left(\frac{\hbar}{\sqrt{2}}\right)2(-i\sqrt{2})\left(e^{i\omega_0 t} - e^{-i\omega_0 t}\right) = \hbar\sin(\omega_0 t)$$

$$\frac{d}{dt}\langle\hat{S}_y\rangle = \frac{i}{\hbar}\langle[\hat{H},\hat{S}_y]\rangle = \frac{i}{\hbar}(-\omega_0)\langle\underbrace{[\hat{S}_z,\hat{S}_y]}_{-i\hbar\hat{S}_x}\rangle$$

$$= -\omega_0\langle\hat{S}_x\rangle$$

integrating,

$$\langle\hat{S}_y\rangle = \hbar\cos(\omega_0 t)$$