Statistics Worksheet
Intermediate Quantum 491

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1. Write a computer program (in, for example, Matlab or python) proving the central limit theorem. Start with a single, flat random number generated between 0 and 1.

2. A general procedure for generating random numbers distributed as the inverse transform method. Given that $f(x)$ is a probability density function (PDF), the cumulative probability $F(a) = \int_0^a f(x)dx$ is a uniform PDF. Choose a uniform random number $0 < u < 1$, then $x = F^{-1}$ will be a random number distributed according to $f(x)$. *(Think about the fundamental theorem of calculus.)* See Figure 1. This assumes that the function F can be inverted analytically. Write a computer program to illustrate this with an exponential decay law.

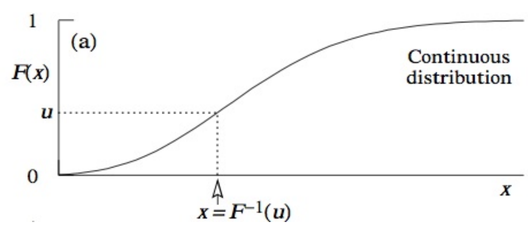

Figure 1: illustration of inverse transform method (from the Particle Data Group).

3. Use the inverse function theorem applied to Gaussian Integrals to show that one can generate Gaussian random numbers by selecting 2 uniform random numbers u_1, u_2 on the interval $(0, 1)$ and then $z_1 = \sin(2\pi u_1)\sqrt{-2\ln u_2}$ and $z_2 = \cos(2\pi u_1)\sqrt{-2\ln u_2}$ will be 2 Gaussian distributed, uncorrelated random numbers with mean 0 and variance 1. $z_i' = \mu + \sigma z_i$ will be Gaussians with mean μ and variance σ^2.

   Write a Gaussian random number generator and make a scatter plot proving that z_1, z_2 are uncorrelated.

4. Generate a set of linearly correlated pseudo data using 2 linearly related variables $y = 2x$ between $x = 0 < x < 10$. Generate the y values so that they are Gaussian distributed about the mean $2x$ with a variance that increases linearly with x: let $\sigma(x) = 0.1\sqrt{x}$. Make a scatter plot of y versus x. Calculate the chi-square $\chi(m)$ with this pseudo-data for a 1 parameter fit to a line with zero intercept and slope m. Plot $\chi(m)$ versus m in the range $1 < m < 4$. Show that the likelihood is chi-square and determine the fit mean and error graphically.