Intermediate Quantum 491: TEST # 1

Please return the test with your work. No book, no notes, calculator OK.

The probability current (prime denotes spatial derivative):

$$j_x = \frac{\hbar}{2mi}\left(\psi^\star \psi' - \psi \psi^{\star\prime}\right)$$

Spinor states:

$$|\pm x\rangle = \frac{1}{\sqrt{2}}\left(|+z\rangle \pm |-z\rangle\right)$$

$$|\pm y\rangle = \frac{1}{\sqrt{2}}\left(|+z\rangle \pm i\,|-z\rangle\right)$$

The spin operators in the z-basis are $\hat{S}_i = (\hbar/2)\hat{\sigma}_i$ where the Pauli spin matricies are: $\hat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$; $\hat{\sigma}_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$; $\hat{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

#1) Consider a particle of mass m in a 1-D box, $0 < x < a$. Suppose that at time $t = 0$ the particle is in the properly normalized state,

$$\psi(x,0) = \sqrt{\frac{2}{a}}\left[\frac{3}{5}\sin\left(\frac{\pi x}{a}\right) + \frac{4i}{5}\sin\left(\frac{3\pi x}{a}\right)\right]$$

a) Determine the probabilities P_1, P_2 and P_3 to measure the ground state energy E_1, the first excited state energy E_2 and the third excited state energy E_3.

b) What is the ground state energy E_1? In the following questions, express all energies in terms of the ground state energy E_1.

c) What is the expectation value $\langle E\rangle$?

d) What is the wave function at time t, $\psi(x,t)$?

#2) Consider a particle of mass m scattering off of a step potential, $V(x) = 0,\ x < 0$ and $V(x) = V_0,\ x > 0$. For $E < V_0$

a) Sketch the wave function (real part) on the figure below.

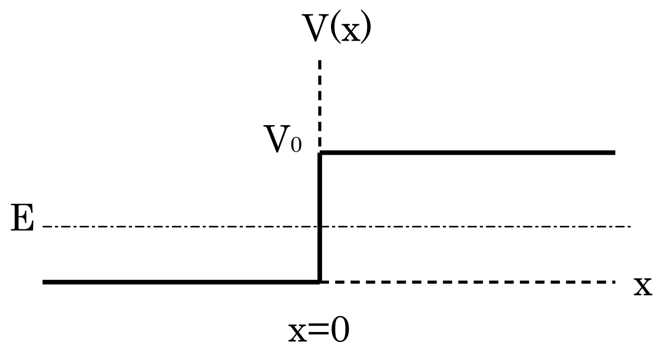
b) Write the wave function for all x in terms of unknown coefficients $(A \equiv 1)\ B, C$

c) Determine B and C. Write the complex number B as $B = |B|e^{i\phi}$. What is the norm $|B|$? You need not bother to determine ϕ. In terms of ϕ, what is C?

d) What are the reflection and transmission coefficients?

e) In the positive x region, what is the chararistic length for the penetration depth? Evaluate it numerically for $E = V_0/2$ and $V_0 = 8$ eV. Use the approximate values $m_e c^2 = \frac{1}{2}10^6$ eV and $\hbar c = 200$ eV·nm.

f) In the limit $V_0 >> E$ what is the wave function for negative x. Explain why you would expect this wave function in this limit.



#3) The spin state corresponding to the eigenvalue $+\hbar/2$ measured along the direction $\hat{r} = \sin\theta\,(\cos\phi\hat{x} + \sin\phi\hat{y}) + \cos\theta\hat{z}$ is

$$|+r\rangle = \cos\frac{\theta}{2}\,|+z\rangle + e^{i\phi}\sin\frac{\theta}{2}\,|-z\rangle$$

The particle in this spin state $|+r\rangle$ is passed through a SG device oriented along $+y$.

a) What are the amplitudes to measure the values $\pm\hbar/2$? The probabilies? Check what you get for $\theta = \pi/2$ and $\phi = 0$? $\theta = \pi/2\ \phi = \pi/2$.

b) What is $\langle S_y\rangle$ and the uncertainty ΔS_y for a particle in the state $|+r\rangle$?