Bose-Einstein Condensate

March 23 2017

The Nobel Prize in Physics 2001 was awarded jointly to Eric A. Cornell, Wolfgang Ketterle and Carl E. Wieman "for the achievement of Bose-Einstein condensation in dilute gases of alkali atoms, and for early fundamental studies of the properties of the condensates".

A system of particles with integral spin obeys Bose-Einstein statistics.

$$f_{BE} = \frac{1}{Ae^{E/k_BT} - 1} = \frac{1}{e^{(E-\mu)/k_BT} - 1}$$

$\mu \equiv$ chemical potential $\mu < 0$ generally a function of temperature (T). Physically, $-\mu \equiv$ energy required to remove one molecule.

$$\frac{dn}{d^3p} = (2s+1)\frac{1}{(2\pi\hbar)^3} f_{BE}(E)$$

The total number density n is fixed,

$$n = (2s+1)\int_0^\infty \frac{4\pi p^2 dp}{(2\pi\hbar)^3} f_{BE}(E)$$

The non-relativistic energy is $E = p^2/2m$ so

$$n = {(2s+1)} \int_0^\infty \frac{4\pi\sqrt{2mE}}{(2\pi\hbar)^3} m f_{BE}(E) dE$$

let $x = E/(k_B T)$

then

$$n = (2s+1)\left(\frac{2mk_BT}{\hbar^2}\right)^{3/2}\int_0^\infty \frac{\sqrt{x}dx}{\exp(x-\mu/(k_BT))-1}$$

Some atoms reach a critical temperature where $\mu(T_c) = 0$ so that

$$n = (2s+1)\left(\frac{2mk_BT}{\hbar^2}\right)^{3/2}\int_0^\infty \frac{\sqrt{x}dx}{e^x-1}$$

The value the dimensionless integral is $I = \frac{\sqrt{\pi}}{2}\zeta(3/2) = 2.32$ using the Zeta function $\zeta(3/2) = 26.12$

We can solve for T_c,

$$T_c = \frac{2\pi\hbar^2}{mk_B}\left(\frac{n}{(2s+1)\zeta(3/2)}\right)^{2/3}$$

Below T_c, Einstein predicted that a fraction of the atoms will collapse into the same quantum state known as a Bose-Einstein condensate.

Below T_c

$$n = n_{cond} + \frac{2s+1}{4\pi^2}(2.32)\left(\frac{2mk_BT}{\hbar^2}\right)^{3/2}$$

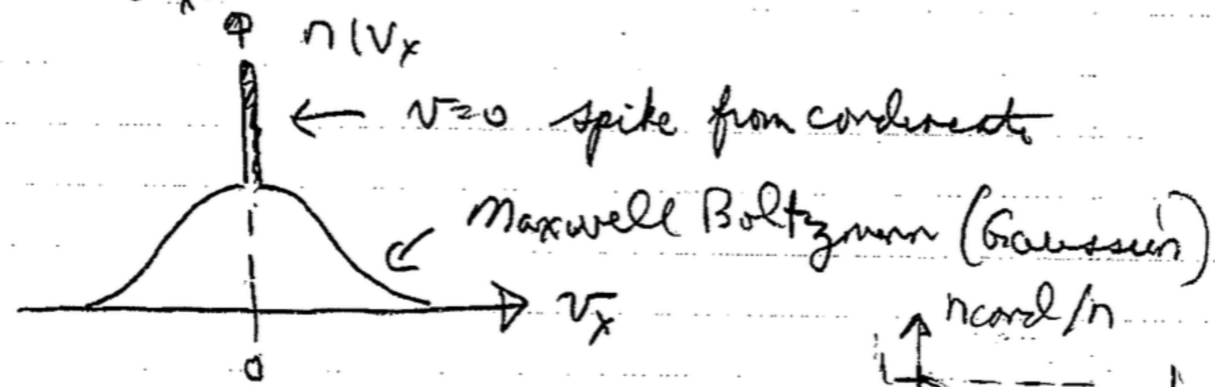
$$n = n_{cond} + n\left(\frac{T}{T_c}\right)^{3/2}$$

Giving for number density of the condensate (n_{cond})

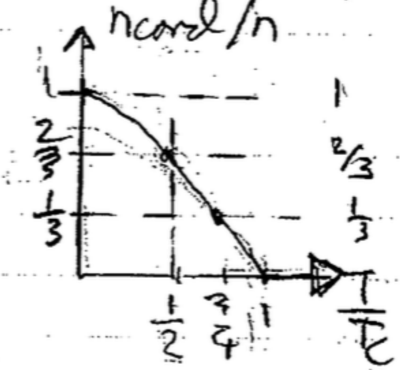
$$n_{cond} = n\left[1 - \left(\frac{T}{T_c}\right)^{3/2}\right]$$

1995, Conclusively observed in dilute gas of ^{87}Rb $\quad T_c = 190 \times 10^{-9}$ K

measured $n(v_x)$



measured $\dfrac{n_{cond}(T)}{n} = 1 - \left(\dfrac{T}{T_c}\right)^{3/2}$



Related phenomena are other types of Condensates (Bosons collapsed into groundstate) superfluidity, superconductivity.

In superfluid liquid helium, He atoms are interacting (chemical potential $\neq 0$), so that entire liquid is in ground state and exhibits zero viscosity.

Atom is Boson if nucleons + electrons is even
^{87}Rb(Z=37) is a Boson

http://www-physics.mps.ohio-state.edu/~perry/p5501_sp16/articles/ketterle-lecture.pdf

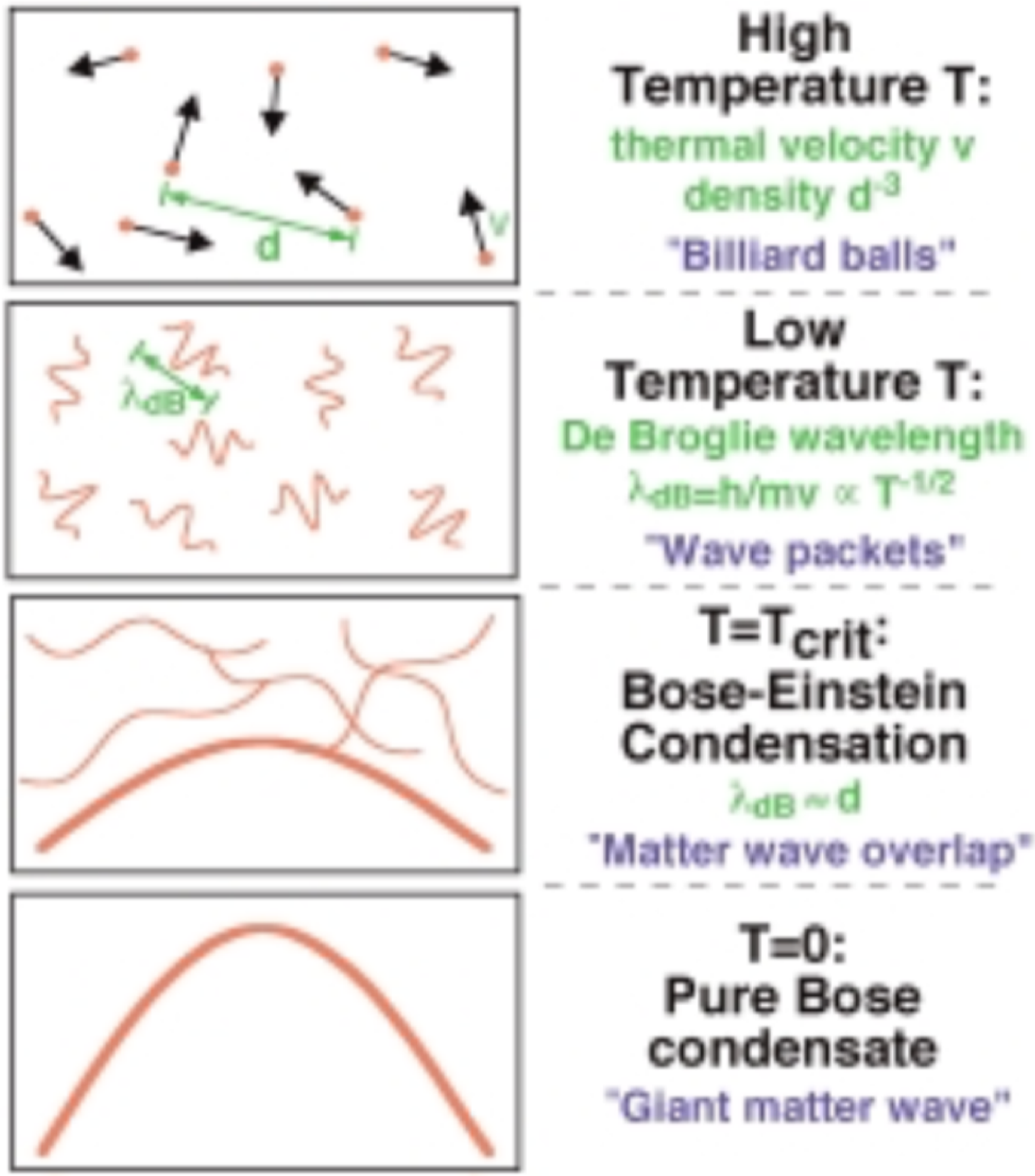
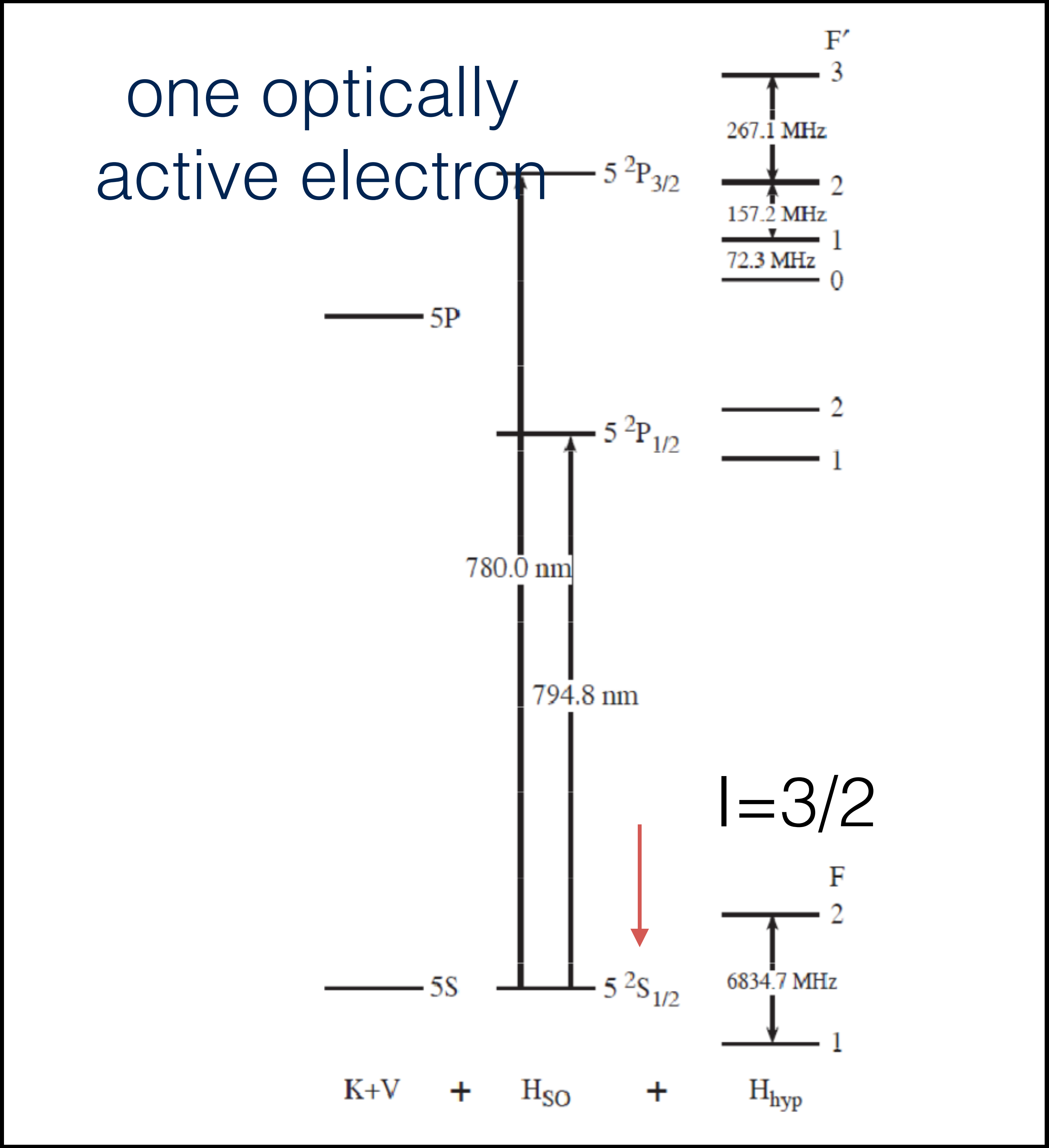


Figure 2. Criterion for Bose-Einstein condensation. At high temperatures, a weakly interacting gas can be treated as a system of "billiard balls." In a simplified quantum description, the atoms can be regarded as wavepackets with an extension of their de Broglie wavelength λ_{dB}. At the BEC transition temperature, λ_{dB} becomes comparable to the distance between atoms, and a Bose condensate forms. As the temperature approaches zero, the thermal cloud disappears, leaving a pure Bose condensate.

37 Rb rubidium : [Kr] $5s^1$



m_F=1,0,-1

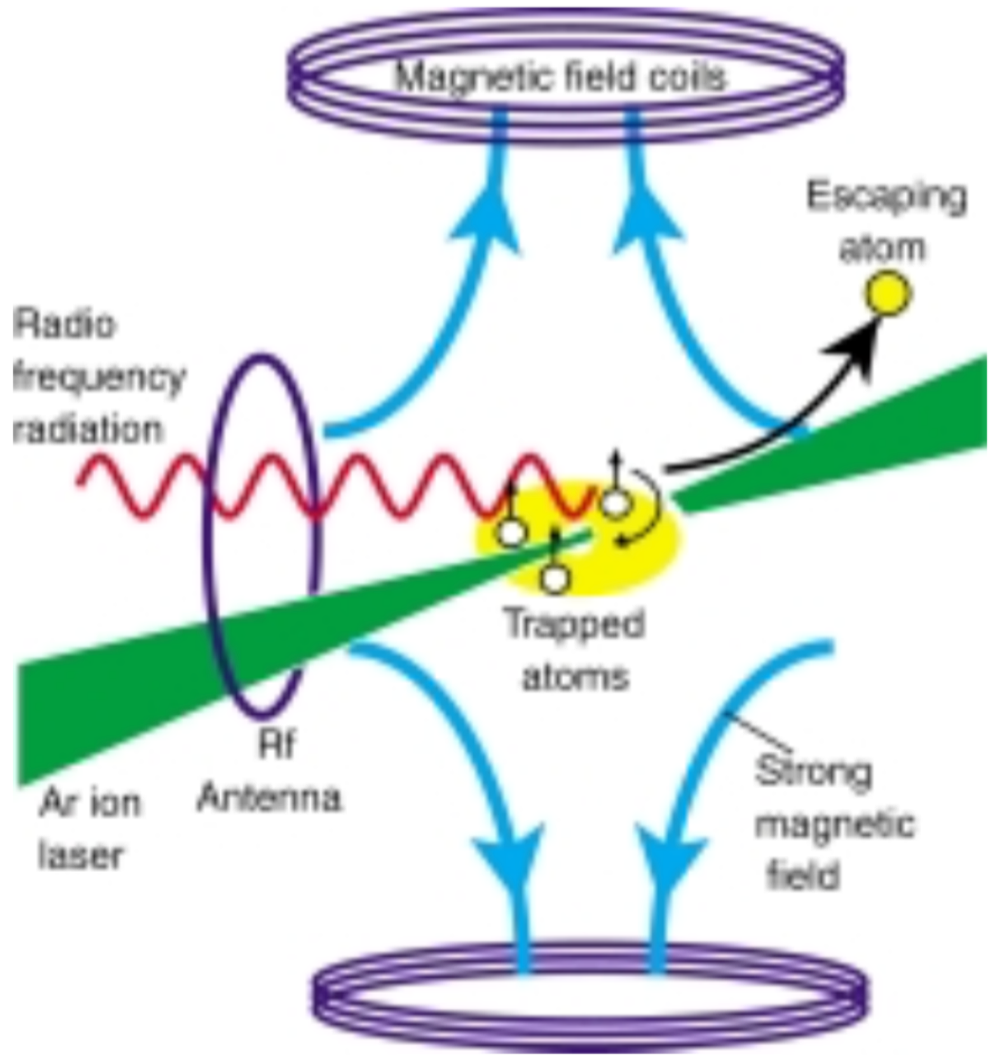


Figure 3. Experimental setup for cooling atoms to Bose-Einstein condensation. Sodium atoms are trapped by a strong magnetic field, generated by two coils. In the center, the magnetic field vanishes, which allows the atoms to spin-flip and escape. Therefore, the atoms are kept away from the center of the trap by a strong (3.5 W) argon ion laser beam ("optical plug"), which exerts a repulsive force on the atoms. Evaporative cooling is controlled by radio-frequency radiation from an antenna. The rf selectively flips the spins of the most energetic atoms. The remaining atoms rethermalize (at a lower temperature) by collisions among themselves. Evaporative cooling is forced by lowering the rf frequency.

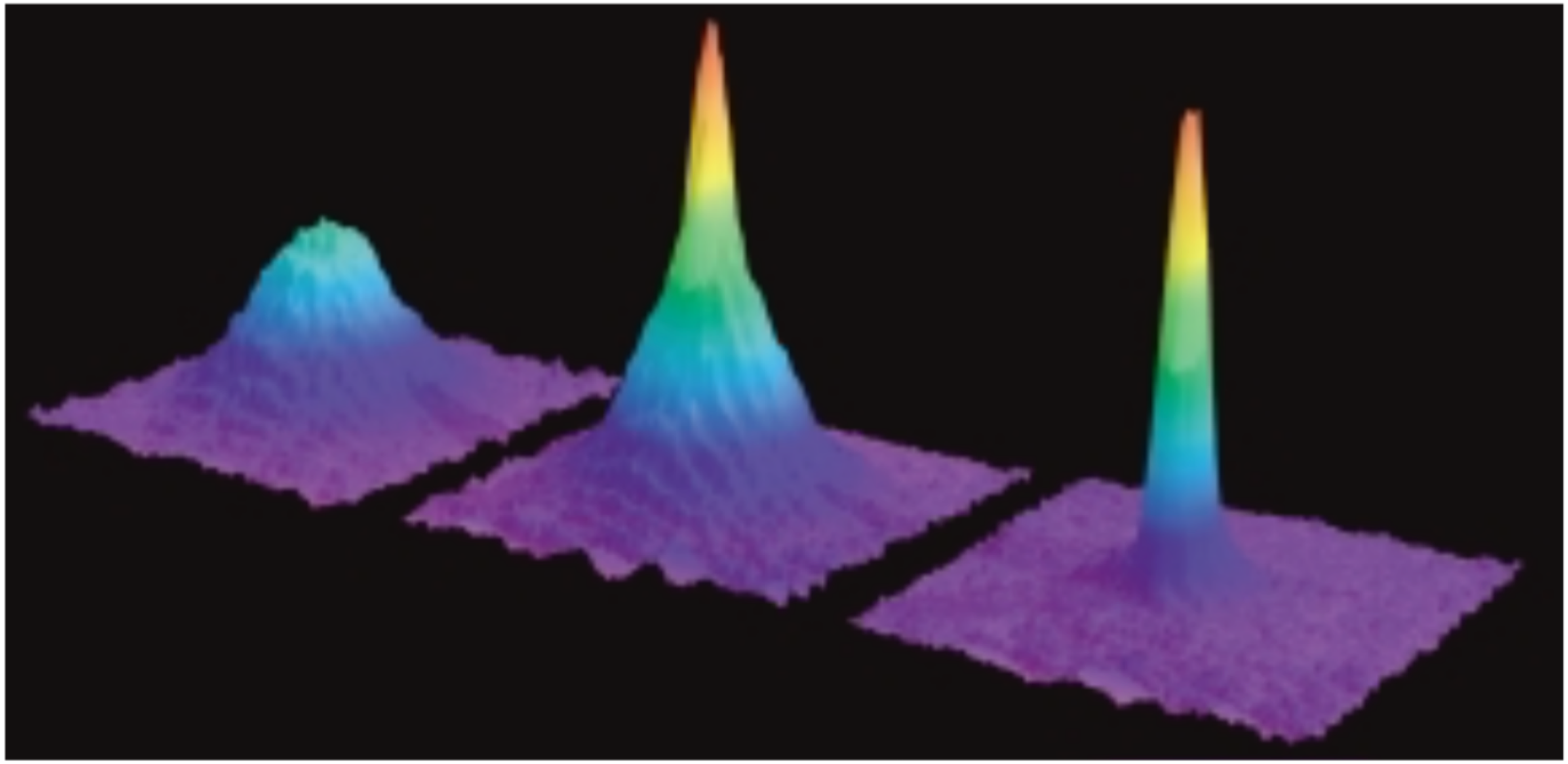
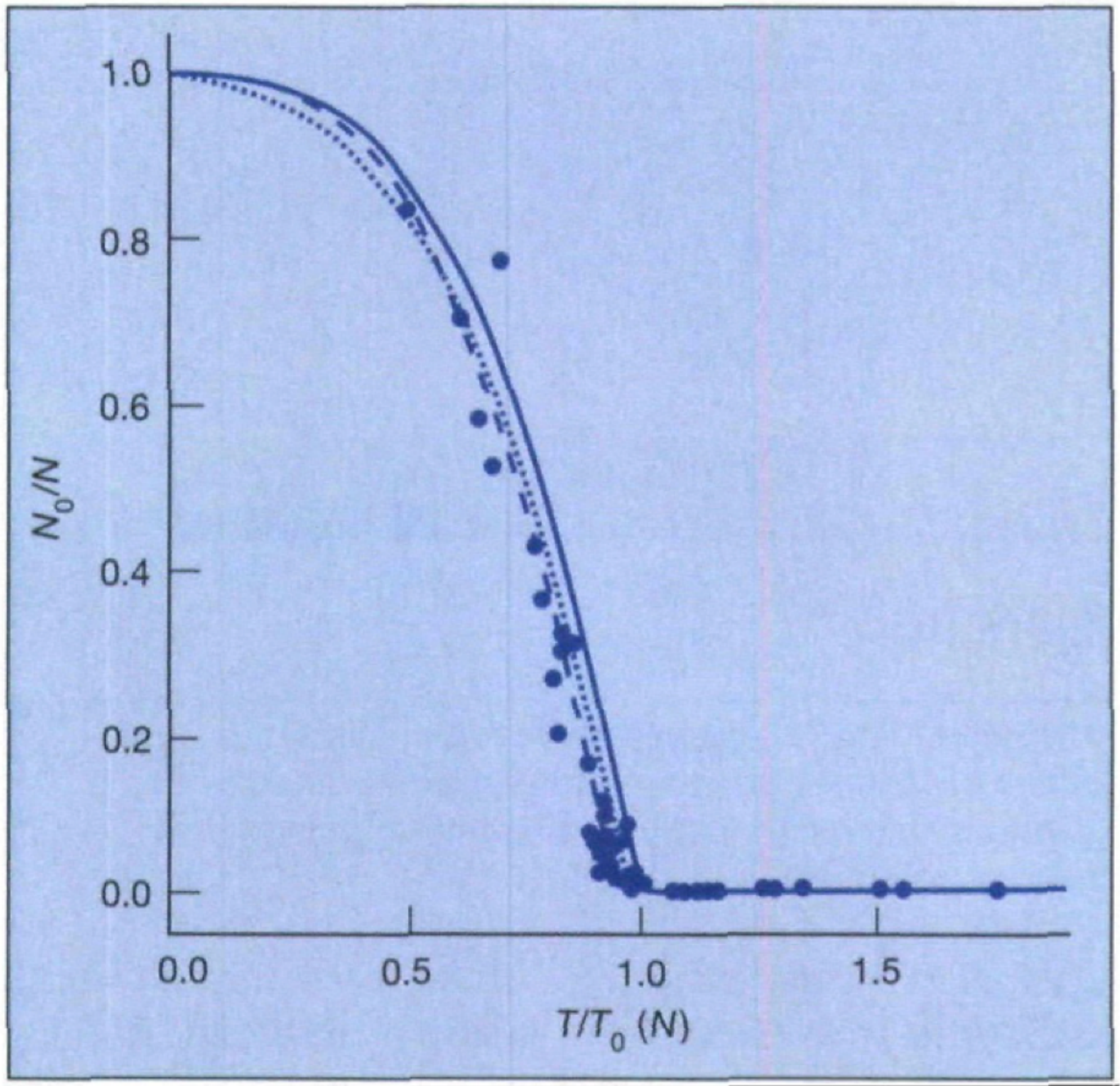


Figure 7. Observation of Bose-Einstein condensation by absorption imaging. Shown is absorption vs. two spatial dimensions. The Bose-Einstein condensate is characterized by its slow expansion observed after 6 ms time-of-flight. The left picture shows an expanding cloud cooled to just above the transition point; middle: just after the condensate appeared; right: after further evaporative cooling has left an almost pure condensate. The total number of atoms at the phase transition is about 7×10^5, the temperature at the transition point is 2 μK.



5 Condensate fraction, N_0/N, measured as a function of scaled temperature, T/T_0, at Boulder. T_0 is the predicted critical temperature for an ideal gas in a harmonic trap in the thermodynamic limit (many atoms). The solid curve is the predicted dependence in the thermodynamic limit, and the dotted curve includes a correction for the finite number of atoms (4000) in the condensate. To the uncertainties in the data, the measurement is consistent with the theory for non-interacting bosons. The dashed line is a best fit to the experimental data.