Intermediate Quantum 492: HW #2 Extra

#1) a) Prove that bound states in one dimension are non-degenerate. Assume that there exists two states ψ_1, ψ_2 with the same energy. Starting from the Schrödinger equation, show that

$$\psi_2 \frac{\partial}{\partial x}\psi_1 \; - \psi_1 \frac{\partial}{\partial x}\psi_2 \; = \text{constant.}$$

Then show that the constant must be zero for bound states. Then prove that ψ_1, ψ_2 differ by an overall (physically irrelevant) phase factor.
b) In three dimensions, there often are degenerate bound states. Where does the proof break down in three dimensions?

#2) Justify the rigid rotor approximation for HCl by calculating the classical turning points of the ground state. Recall the effective spring constant $k = 30eV/\text{Å}^2$ and $r_0 = 1.27\text{Å}$

#3) Prove the operator identity

$$\left(\frac{1}{r}\frac{\partial}{\partial r}r\right)^2 = \frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r}.$$

Show that the substitution of R=u/r reduces the radial equation in three dimensions to a form like the one dimensional Schrödinger equation.
#4) The naive choice for a radial momentum operator is,

$$\hat{p}_N = \frac{1}{r}\vec{r} \cdot \hat{\vec{p}}$$

Show that $\hat{p}_N$ is not Hermitian. Define the radial momentum operator as

$$\hat{p}_r = \frac{1}{2}\left(\hat{p}_N + \hat{p}_N^\dagger\right),$$

and show that in position space it is represented by

$$\hat{p}_r = \frac{\hbar}{i}\frac{1}{r}\frac{\partial}{\partial r}r$$