Intermediate Quantum 492: HW #3 Extra

Prove Kramer's relation for expectation values of r to power s for the electron wave functions ψ_{nlm} of the hydrogen atom:

$$\alpha\langle r^s\rangle + \beta a_0\langle r^{s-1}\rangle + \gamma a_0^2\langle r^{s-2}\rangle = 0$$

where $\alpha = (s+1)/n^2$. $\beta = -(2s+1)$ and $\gamma = (s/4)\left[(2\ell+1)^2 - s^2\right]$

start with the radial equation. then expectation values $\langle r^s\rangle = \int_0^\infty r^s u(r)^2 dr$ so multiply equation by ur^s and integrate. get a second equation by multiplying the radial equation by $u'r^{s+1}$ and integrate (u' is the derivative with respect to r) . everything else follows by integrating by parts. additional (different) hints can be found in Griffith's problem 6.34 page 288 (2^{nd} edition).