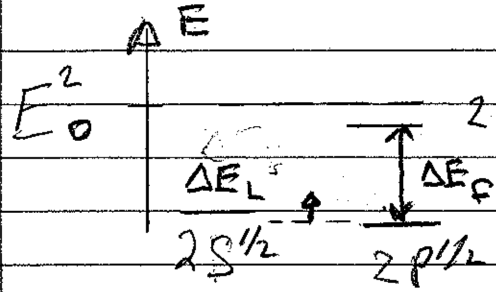
Physics 492 — Spring 2010
M. Gold

Lecture 12: Lamb Shift



$\Delta E_f = 10.9\,\text{GHz}$

$\Delta E_L = 1057\,\text{MHz}$

Lamb shift

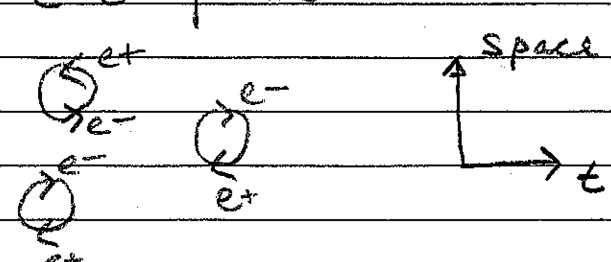
Relativistic Q.M. cannot be single particle theory. To account for particle/antiparticle creation/annihilation must be quantum field theory. For electromagnetism called quantum electrodynamics (QED).

QED is physical origin of Darwin term.

Quantized field is collection of harmonic oscillator states for each mode (wave number $\vec{k}$). Excitations are particles. Ground state (vacuum) has zero-point energy.

Vacuum has quantum fluctuations of "virtual" e^+e^- pairs

$|\phi\rangle$:



"e^+ like e^- moving backwards in time"

"Virtual" means violating $E, \vec{p}$ conservation as allowed by uncertainty principle.

$$\Delta E \Delta t \sim \hbar$$

$$\Delta E = 2mc^2, \quad \Delta t \sim \frac{\hbar}{2mc^2}$$

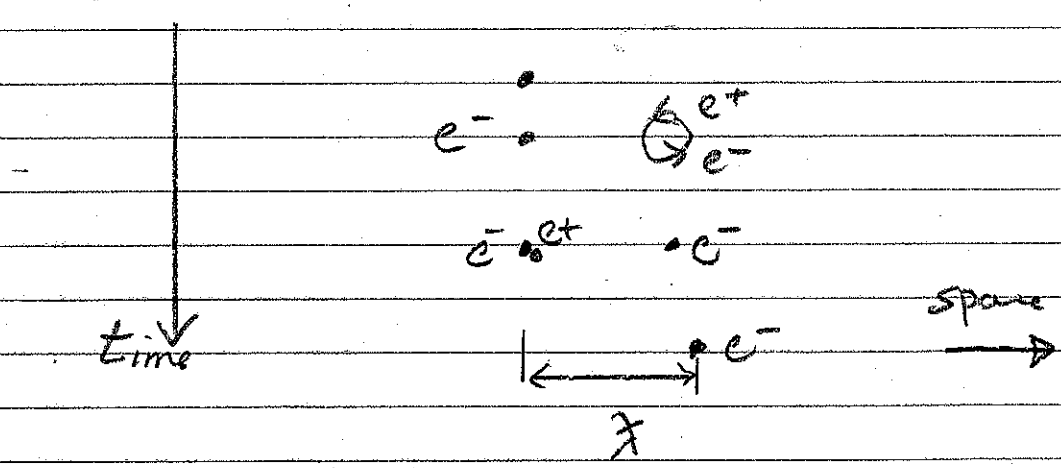
Virtual e^+e^- can separate to distance

$$e^+ \longleftarrow\!\!+\!\!\longrightarrow e^-$$

$$\Delta x = 2c\Delta t = \frac{\hbar}{mc} = ƛ$$

where $ƛ$ = compton wavelength $/2\pi$

Physical limitation on localization of electron. Imagine time sequence:

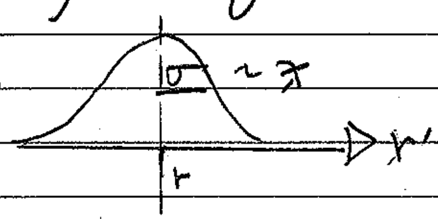


Think of this as zitterbewegung (German for quivering motion)

In terms of e^- proton relative coordinate $\vec{r}$, zitterbewegung smears $V(r)$.

$$\tilde{V}(\vec{r}) = \int d^3\vec{r}' \, f(\vec{r}') \, V(\vec{r}+\vec{r}')$$

$f(r')$



Effect is calculated in QED, but we can get form of effective hamiltonian correction by using Gaussian $f(r')$.

expand $V(\vec{r}+\vec{r}') = V(\vec{r}) + \vec{r}'\cdot\vec{\nabla}V\big|_{\vec{r}}$

$$+ \frac{1}{2}\left(\vec{r}'\cdot\vec{\nabla}\right)^2 V\big|_{\vec{r}} + \dots$$

$$\langle \vec{r}'\rangle_f = \int d^3\vec{r}' f(r')\, r' = 0$$

then integrating over f:

$$\tilde{V}(\vec{r}) = V(\vec{r}) + \frac{1}{2}\sum_{i,j=1}^{3} \langle x_i x_j\rangle_f \frac{\partial^2 V}{\partial x_i \partial x_j}$$

$x_i = (x, y, z)$

cross terms $\langle xy\rangle$ are zero.

$\langle x_i^2\rangle_f$ are all equal to $\frac{1}{3}\langle r^2\rangle_f$

$$\tilde{V}(\vec{r}) = V(\vec{r}) + \frac{1}{2}\cdot\frac{1}{3}\langle r^2\rangle_f \sum \frac{\partial^2 V}{\partial x_i^2}$$

$$= V(\vec{r}) + \frac{1}{6}\langle r^2\rangle_f \nabla^2 V$$

Correction is

$$H' = \frac{1}{6}\langle r^2\rangle_G \nabla^2 V = \frac{\lambda^2}{6}\nabla^2 V$$

where we have used Gaussian $\langle r^2\rangle_G = \sigma^2 = \lambda^2$

This is same form as Darwin term

$$\hat{H}_D = \frac{\lambda^2}{8}\nabla^2 V$$

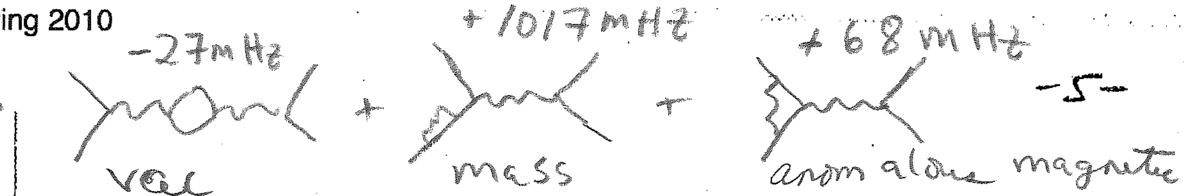
Lamb shift is due to fluctuations in $\vec{\mathcal{E}}$ from vacuum fluctuations. mostly effective $+\delta^3(\vec{r})$ term raising $l=0$ states. A small vacuum polarization term offsets $+\delta^3(\vec{r})$ term by -27 MHz.

QED vacuum is polarized by virtual e^+e^- pairs effectively shielding interaction of "naked" or "bare" charge.

$$\Delta E_L\left(2S_{\frac{1}{2}} - 2P_{\frac{1}{2}}\right) = (\text{const})\,\frac{mc^2 (Z\alpha)^4}{n^3}\,\alpha \ln\left(\frac{1}{Z\alpha}\right)$$

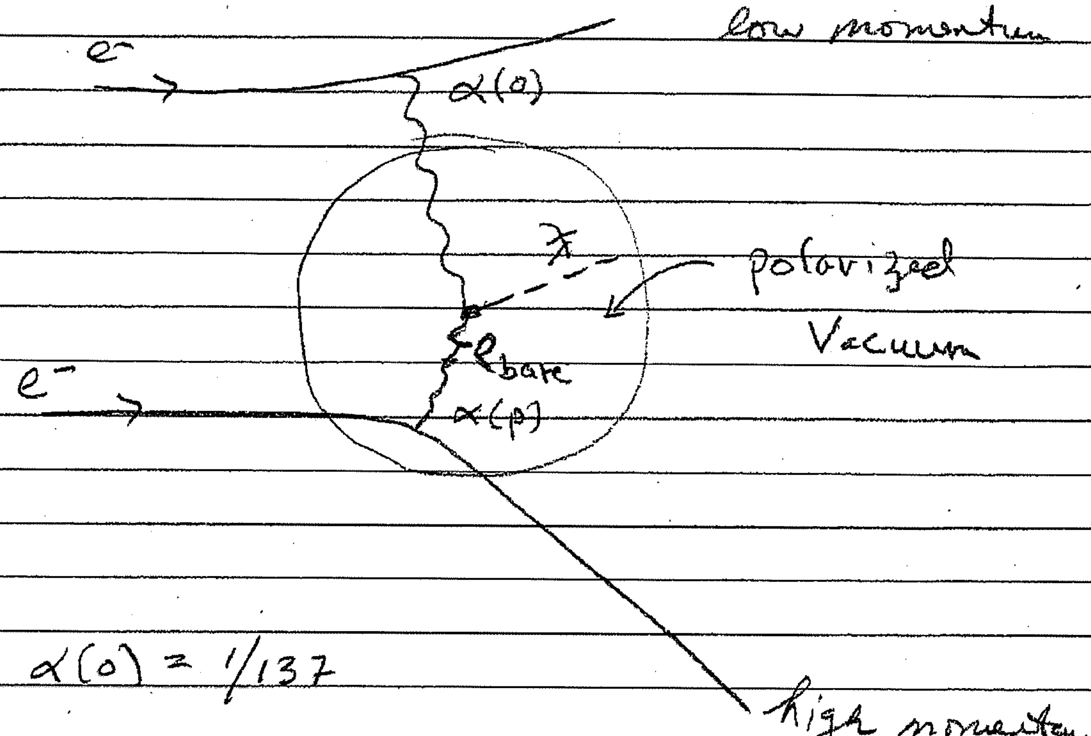
for $Z=1$, $= 1/28$

in QED:



picture of QED Vacuum polarization

$e^-e^- \to e^-e^-$



$\alpha(0) = 1/137$

$\alpha(p) > \alpha(0)$

for example, for $p = m_Z c = 91$ GeV/c

$$\alpha(m_Z c) = 1/128$$

- Lamb shift is precision test of QED

PRL 68 1120 (1992)

$\Delta E_L^{exp}/h \, (2S-2P) = (1057.845 \pm 0.009)$ MHz

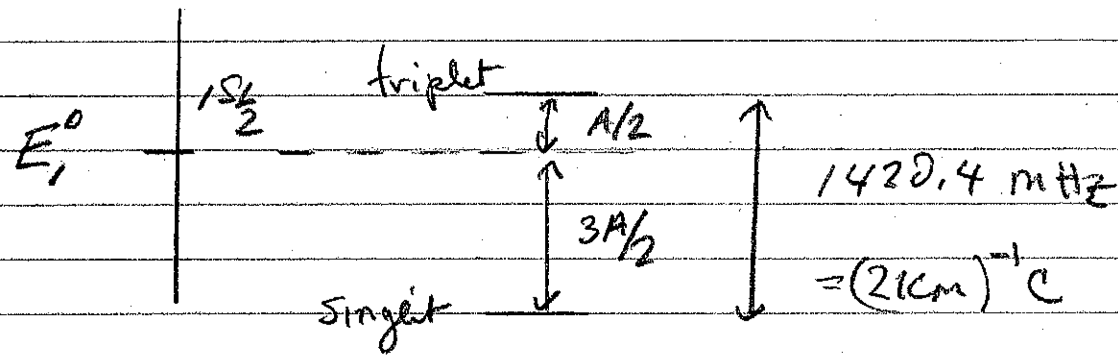
$\Delta E_L^{th}/h \, (2S-2P) = (1057.874 \pm 0.018)$ MHz

Theory limited by uncertainty in $\langle r \rangle$ proton (exp).
Exp. limited by (irreducible) 2p natural line width.

Hyperfine splitting

Effect of nuclear spin lifts 4-fold degeneracy of ground state

$$\hat{H}_{hf} = (\text{constant})\, \vec{\mu}_e \cdot \vec{\mu}_p\, \delta^3(\vec{r}\,) + (\ell \neq 0 \text{ term})$$



$$\vec{S} = \vec{S}_e + \vec{S}_p$$

$$A = \frac{2}{3} mc^2 (Z\alpha)^4 \boxed{\left(\frac{m_e}{m_p}\right) g_p}$$

g_p is proton anomalous magnetic moment (~5.59)

$$\frac{m_e}{m_p} = \frac{1}{1800}$$