Physics 492 Spring 2017

Lecture 7: Nuclear Models

Magic numbers & Shell model

1948 Maria Goeppert Mayer (Nobel 1963)

Certain values of Z or # neutrons = A − Z
are particularly stable ("magic")

2, 8, 20, 28, 50, 82, 126

Some are doubly magic ^4_2He, $^{16}_8O$

Suggests shell structure with filled shells
particularly stable

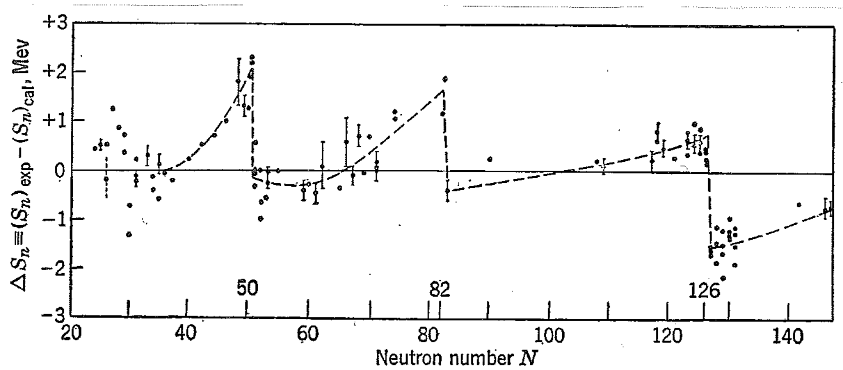


Fig. 3.6 Observed neutron separation energies $(S_n)_{exp}$ compared with $(S_n)_{cal}$ predicted by the smooth variation of the semiempirical mass formula, using Fermi 1945 coefficients (Table 3.3) in Eq. (3.65). Discontinuities of the order of 2 Mev are evident for $N = 50$, 82, and 126. Evidence for a shell closure at $N = 28$ is inconclusive. [*Adapted from Harvey* (H22).]

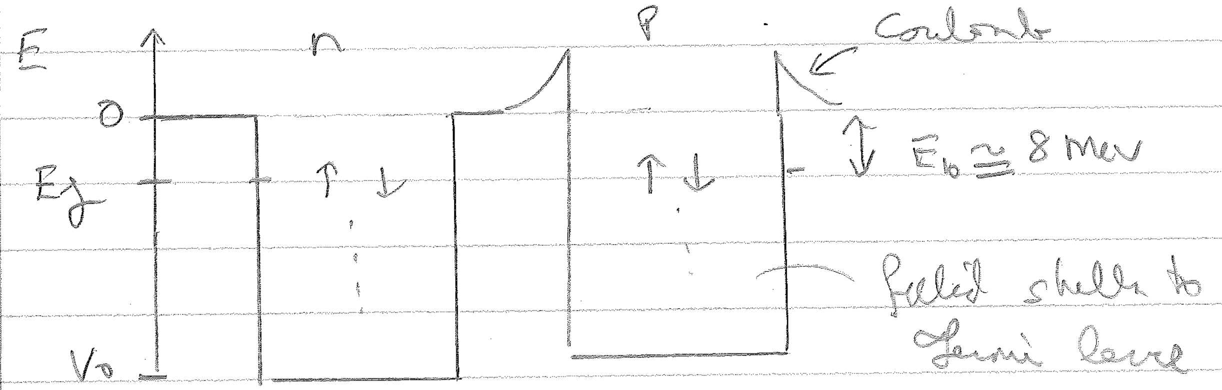
Neutron separation energies
compared to smooth variation predicted
by semi-emperical mass formula.

from Evans, The Atomic nucleus, McGraw Hill 1955

<u>Single particle model</u> ($N \equiv$ nucleon, p or n)

N-N interaction in nuclear ground state (filled shells) nearly absent due to exchange symmetry.
Assume remaining N move in effective potential of other nucleons

<u>Fermi-Gas model</u> & estimate of Depth of nuclear well



Cartoon of model

$$E_b + E_f = V_0$$

$$E_f = \frac{p_F^2}{2m} \quad \text{Fermi momentum}$$

Number density in quantum phase space

$$dN = V \frac{d^3p}{h^3} \quad \text{where } V \text{ is nuclear volume}$$

$$V = \frac{4\pi}{3} r^3 = \frac{4\pi}{3}\left(A^{1/3} r_0\right)^3 = \frac{4\pi}{3} A r_0^3$$

For degenerate gas, states are filled to E_F.

$$\int d^3p = 4\pi \int_0^{P_F} p^2 dp = \frac{4\pi}{3} P_F^3$$

total Number for neutrons or protons

$$N = \frac{V}{(2\pi\hbar)^3} \left(\frac{4\pi}{3} P_F^3 \right) \times \underset{\text{spin}}{2}$$

$$= \left(\frac{1}{2\pi\hbar}\right)^3 \left(\frac{4\pi}{3}\right)^2 A (r_0 P_F)^3$$

$$= \frac{4}{9\pi} A \left(\frac{r_0 P_F}{\hbar}\right)^3$$

for heavy nucleus take $Z \simeq A - Z$

$$N = Z = \frac{A}{2} = \frac{4}{9\pi} A \left(\frac{r_0 P_F}{\hbar}\right)^3$$

then P_F is independent of A

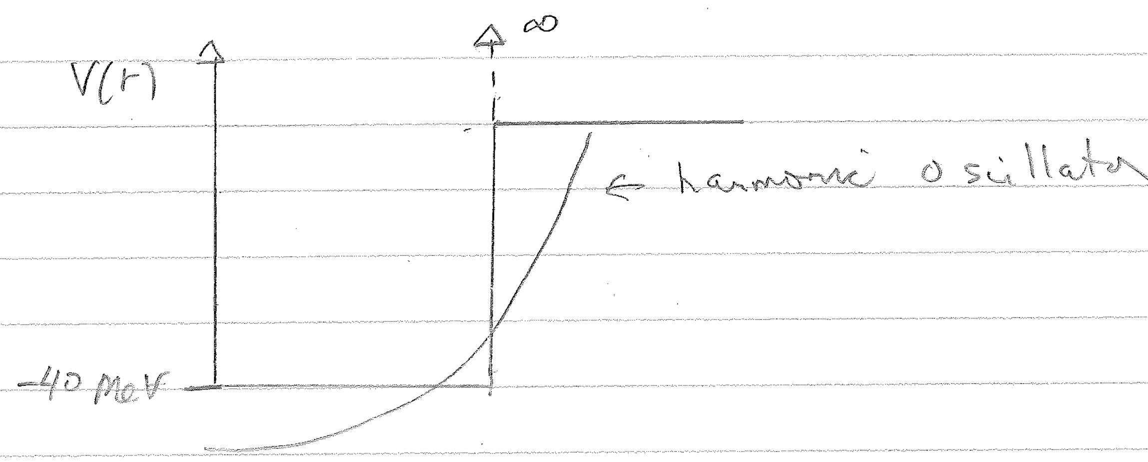
$$P_F = \frac{\hbar}{r_0} \left(\frac{9\pi}{8}\right)^{1/3}$$

$$E_F = \frac{P_F^2}{2m_N} = \frac{1}{2\underbrace{m_N c^2}_{940 \text{ MeV}}} \left(\frac{\overbrace{\hbar c}^{197 \text{ MeV fm}}}{\underbrace{r_0}_{1.2 \text{ fm}}}\right)^2 \left(\frac{9\pi}{8}\right)^{2/3} = 33 \text{ MeV}$$

Depth of well estimate is

$$V_0 = E_b + E_F \cong (8 + 33) \text{ MeV} = \underline{\underline{41 \text{ MeV}}}$$

Nuclear Potential Models



In general, l is not zero.

Infinite square well $\quad \Psi_{nl} = R_{nl} Y_{lm}$

$$\frac{d^2 R}{dr^2} + \frac{2}{r}\frac{dR}{dr} - \frac{l(l+1)}{r^2} R + k^2 R = 0$$

$$k \equiv \sqrt{2\mu E}/\hbar$$ note that for infinite well, $E > 0$.

define $\rho \equiv kr$, $R' = dR/d\rho$

$$R'' + \frac{2}{\rho} R' + \left(1 - \frac{l(l+1)}{\rho^2}\right) R = 0$$

Series solution yields spherical Bessel function

$$j_l(\rho) = (-\rho)^l \left(\frac{1}{\rho}\frac{d}{d\rho}\right)^l \frac{\sin\rho}{\rho}$$

and Neuman function $n_l(\rho)$ but not finite at $\rho = 0$ and therefore do not apply.

Spherical Bessel functions:

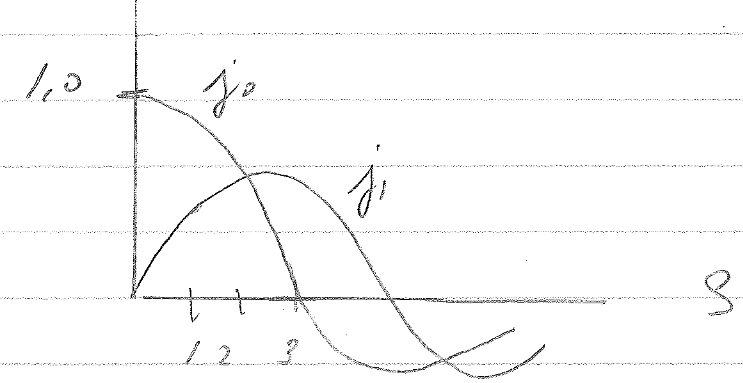
$$j_0(\rho) = \frac{\sin\rho}{\rho}$$

for $l > 0$ $\quad j_l(\rho) \underset{\rho \to 0}{\sim} \frac{\rho^l}{\prod_{m=0}^{l}(2m+1)}$

$$j_l(\rho) \underset{\rho\to\infty}{\sim} \frac{1}{\rho} \cos\left[\rho - \frac{\pi}{2}(l+1)\right]$$

(these functions also describe diffraction through circular aperture)

Sketch:



__Energy__ determined by $j_l(ka) = 0$
boundary condition

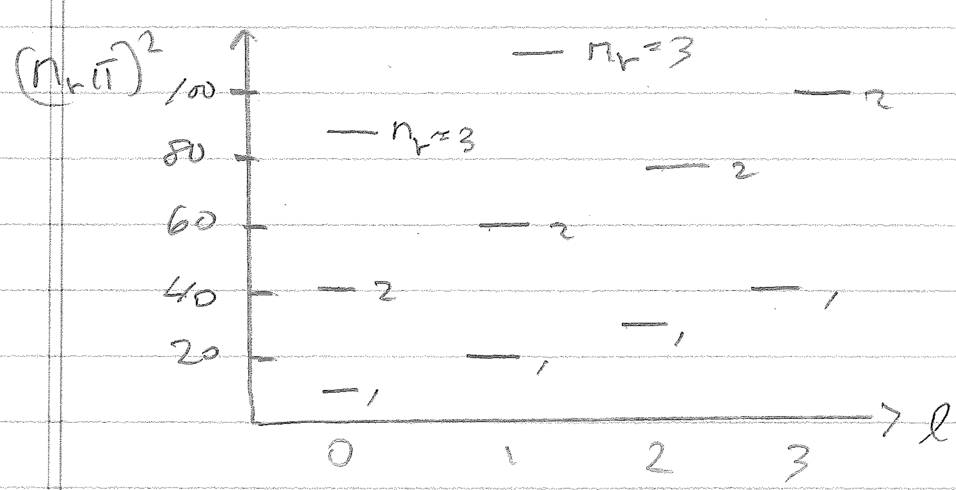
for $l = 0$

$$j_0(ka) = \frac{\sin ka}{ka} = 0$$

gives $ka = n_r \pi$ n_r also counts number of node in j_0

$$E_{n,0} = \frac{\hbar^2 k^2}{2\mu} = \frac{\hbar^2}{2\mu}\left(\frac{n_r \pi}{a}\right)^2$$

Energy spectrum : $(n_r \pi)^2 = \frac{2\mu E}{\hbar^2} a^2$



note Energy is l dependent

$d = 2(2l+1) = \underline{\underline{2}}$ $\underline{\underline{6}}$ 10 14

closed shells 2, 8
first 2 magic numbers