

Physics 492: Recitation #11
April 13, 2017

1. Consider a two state system $|1\rangle, |2\rangle$ with the unperturbed Hamiltonian

$$\hat{H} = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix}$$

and perturbation

$$\hat{H}_1 = \begin{pmatrix} 0 & \epsilon \\ \epsilon & 0 \end{pmatrix}$$

Find the first non-vanishing perturbative correction. Explain what happens if $\epsilon > |E_2 - E_1|$

2. Show that for an incoming plane wave and an outgoing spherical wave, $d\sigma/d\Omega = |f|^2$. Recall the probability current is $\vec{j} = (\hbar/\mu)\text{Im}(\psi^*\vec{\nabla}\psi)$.
3. Consider elastic scattering by a spherical delta-function potential,

$$v(r) = \frac{\hbar^2\gamma}{2\mu}\delta(r - a)$$

where $q = 2k \sin(\theta/2)$. In the Born approximation find the differential cross section, and, in the low energy limit, the total cross section.

$$f^{(1)} = \frac{-2\mu}{\hbar^2 q} \int_0^\infty r dr \sin(qr)V(r)$$

Find the criteria for the validity of the Born approximation starting from the general criteria,

$$\frac{2\mu}{\hbar^2 k} \left| \int_0^\infty dr e^{ikr} \sin(qr)V(r) \right| \ll 1$$