Physics 492: Recitation #11a
April 20, 2022

1. Consider a perturbation to the hydrogen atom, $H' = \gamma/r$ where γ is a small constant.

   (a) Argue that the energy should depend only on the principle quantum number n and therefore can be written as $\mathrm{E_n}(\gamma) = \langle \mathrm{n}\ell\mathrm{m}|\mathrm{H}^0 + \mathrm{H}'|\mathrm{n}\ell\mathrm{m}\rangle$

   (b) Show that the exact answer is $\mathrm{E_n} = \mathrm{E_n^0}\left(1 - \gamma/\mathrm{Ze}^2\right)^2$ where $E_n^0 = -\mu Z^2 e^4/2\hbar^2 n^2$ is the unperturbed hydrogen energy.

   (c) Show that perturbation theory implies that the first perturbation correction

   $$\gamma E_n^1 = \gamma \left(\frac{dE_n}{d\gamma}\right)_{\gamma=0}$$

   (d) Show that

   $$\left\langle \mathrm{n}\ell\mathrm{m}\left|\frac{1}{\mathrm{r}}\right|\mathrm{n}\ell\mathrm{m}\right\rangle = \frac{\mu \mathrm{Ze}^2}{\hbar^2 \mathrm{n}^2}$$

2. Consider elastic scattering by a spherical delta-function potential,

   $$v(r) = \frac{\hbar^2\gamma}{2\mu}\delta(r-a)$$

   where $q = 2k\sin(\theta/2)$.

   In the Born approximation, the scattering amplitude is

   $$f^{(1)} = \frac{-2\mu}{\hbar^2 q}\int_0^\infty r dr \sin(qr) V(r)$$

   (a) Find the differential cross section.

   (b) Show that in the low energy limit the differential cross section is isotropic and independent of k.

   (c) Find the criteria for the validity of the Born approximation starting from the general criteria,

   $$\frac{2\mu}{\hbar^2 k}\left|\int_0^\infty dr e^{ikr}\sin(qr)V(r)\right| << 1$$