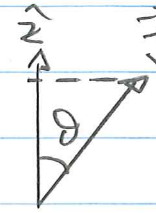
Physics 492 Recitation #2 1/29/2016

#1




"vector model"

$$\cos\theta = \frac{\langle J_z \rangle}{\sqrt{\langle \hat{J}^2 \rangle}} = \frac{j}{\sqrt{j(j+1)}}$$

Spin-$\frac{1}{2}$ $\quad \cos\theta_{1/2} = \frac{1/2}{\sqrt{\frac{1}{2}(\frac{1}{2}+1)}} = \frac{1}{\sqrt{3}} \qquad \theta_{1/2} = 55^\circ$

Spin-1 $\quad \cos\theta_1 = \frac{1}{\sqrt{2}} \qquad \theta_1 = 45^\circ$

macroscopic top $\quad L = I\omega$

$$\sqrt{l(l+1)} \approx l = \frac{L}{\hbar} \qquad \left.\begin{array}{l} m = 10\,g \\ r = 1\,cm \\ \omega = 10^4 \end{array}\right\} L = 10^{-2}\,kg\,m^2/s$$

$$l = \frac{10^{-2}}{10^{-34}} = 10^{32} \qquad \theta_{macro} = 0$$

#2 $\quad \hat{A}|a,b\rangle = a|a,b\rangle$

$\hat{B}|a,b\rangle = b|a,b\rangle$

since $|a,b\rangle$ are complete, $|\psi\rangle = \sum_{i,j} c_{ij}|a_i, b_j\rangle$

$$[\hat{A},\hat{B}]|\psi\rangle = \sum_{i,j} c_{ij}(a_i b_j - b_j a_i)|a_i, b_j\rangle$$

$$= 0$$

therefore as an operator equality, $[\hat{A},\hat{B}] = 0$

#3 $\hat{L}_\pm |l,m\rangle = \hbar\sqrt{l(l+1) - m(m\pm 1)}\, |l, m\pm 1\rangle$

$l = 2 \qquad \sqrt{2 - m(m\pm 1)} =$

| + | − | m = |
|---|---|---|
| 0 | $\sqrt{2}$ | 1 |
| $\sqrt{2}$ | $\sqrt{2}$ | 0 |
| $\sqrt{2}$ | 0 | −1 |

$$\langle m', l | \hat{L}_+ | m, l\rangle = \hbar\begin{pmatrix} 0 & \sqrt{2} & 0 \\ 0 & 0 & \sqrt{2} \\ 0 & 0 & 0 \end{pmatrix}$$

$$\langle m', l | \hat{L}_- | m, l\rangle = \hbar\begin{pmatrix} 0 & \sqrt{2} & 0 \\ 0 & 0 & \sqrt{2} \\ 0 & 0 & 0 \end{pmatrix}^+ = \hbar\begin{pmatrix} 0 & 0 & 0 \\ \sqrt{2} & 0 & 0 \\ 0 & \sqrt{2} & 0 \end{pmatrix}$$

$\hat{L}_\pm = \hat{L}_x \pm i\hat{L}_y$

$$\hat{L}_x = \frac{1}{2}(\hat{L}_x + \hat{L}_y) \qquad \hat{L}_y = \frac{1}{2i}(L_x - \hat{L}_y)$$

$$= \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \qquad = \frac{\hbar}{i\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$$

$$\langle m', l | \hat{L}_z | m, l\rangle = \hbar\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$