Phys 492 Spring 2021

Recitation #4 Solutions

#1) $V_c(r) = -\frac{\alpha \hbar c}{r}$

at nuclear scale $r \approx 1\,fm$

$$|V_c| = \frac{1}{137}\,\frac{197\,MeV\cdot fm}{1\,fm} = 1.4\,MeV$$

Coulomb repulsion $\approx 20\%$ nuclear binding

#2) $\psi = (space)(spin)(isospin)$

identical fermions ψ must be antisymmetric

$(space) = \frac{u}{r} Y_{00}$ for ground state
symmetric

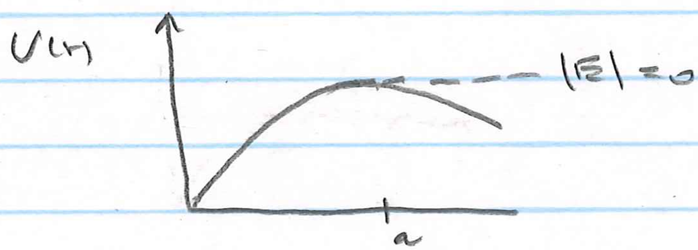
(spin) for $J=1$ is ~~antisymmetric~~

so (Isospin) must be $I=0$ symmetric

(#3) let $\psi = \frac{u}{r} Y_{\ell m}$

$$-\frac{\hbar^2}{2\mu} u'' + \left[\frac{\ell(\ell+1)\hbar^2}{2\mu r^2} + V \right] u = E u$$

ground state has $\ell = 0$ $\quad V = \begin{cases} V_0 & r < a \\ 0 & r > a \end{cases}$



with $Q = \sqrt{2\mu(V_0 - |E|)}/\hbar$

$$q = \sqrt{2\mu |E|}/\hbar$$

with $Q^2 + q^2 = 2\mu V_0/\hbar^2$ $\qquad$ solution is

$$u(r) = \begin{cases} A \sin Qr & r < a \\ C e^{-qr} & r > a \end{cases}$$

Existence of bound state; $\left. \frac{d}{dr}(A \sin Qr) \right|_{r=a} > 0$

$$Qa > \frac{\pi}{2}$$

just barely bound $|E| \ll V_0$

$$\frac{2\mu V_0}{\hbar^2} a^2 > \frac{\pi^2}{4} \; ; \; \boxed{V_0 > \left(\frac{\hbar^2}{2\mu}\right) \frac{\pi^2}{4a^2}}$$