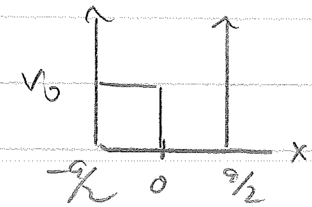
Physics 492 Spring 2017

Recitation #5 Solutions

(1)



$$\phi_n = \begin{cases} \sqrt{\frac{2}{a}} \cos\left(\frac{n\pi x}{a}\right) & n \text{ odd} \\ \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right) & n \text{ even} \end{cases}$$

$$E_1^0 = \frac{\hbar^2}{2m}\left(\frac{\pi}{a}\right)^2 \qquad E_n^0 = n^2 E_1^0$$

$$E_n = E_n^0 + E_n^{(1)} \qquad E_n^{(1)} = V_0 \int_{-a/2}^{0} |\phi_n|^2 \, dx$$

$$n \text{ odd} \quad E_n^{(1)} = V_0\left(\frac{2}{a}\right) \int_{-a/2}^{0} \cos^2\left(\frac{n\pi x}{a}\right) dx$$

$$y = \frac{\pi x}{a} \qquad = V_0 \frac{2}{\pi} \int_{-\pi/2}^{0} \cos^2(ny)\, dy$$

$$= V_0\left(\frac{2}{\pi}\right)\left[\frac{y}{2} + \frac{\sin(2ny)}{4n}\right]_{-\pi/2}^{0} = +\frac{V_0}{2}$$

Same for n-even

recitation #5

(2) $\sum_m \langle n|\hat{H}_1|m\rangle\langle m|\hat{H}_1|n\rangle = \langle n|\hat{H}_1^2|n\rangle$

completeness

(3) $E_0 \pm \sqrt{A^2+(\mu\varepsilon)^2} \underset{A/\mu\varepsilon \ll 1}{\simeq} E_0 \pm \mu\varepsilon\left[1+\frac{1}{2}\left(\frac{A}{\mu\varepsilon}\right)^2\right]$

$= E_0 \pm \mu\varepsilon \pm \frac{1}{2}\frac{A^2}{\mu\varepsilon}$

First non-zero term is quadratic in perturbation. First perturbative term is zero.

$|1\rangle = \begin{pmatrix}1\\0\end{pmatrix}$; $|2\rangle = \begin{pmatrix}0\\1\end{pmatrix}$

$\hat{H}_0 = \begin{pmatrix} E_0+\mu\varepsilon & 0 \\ 0 & E_0-\mu\varepsilon \end{pmatrix}$ $\hat{H}' = \begin{pmatrix} 0 & -A \\ -A & 0 \end{pmatrix}$

$E_n^{(1)} \langle n|\hat{H}'|n\rangle = 0$ for $n = 1, 2$

$E_n^{(2)} = \frac{|\langle k|\hat{H}_1|n\rangle|^2}{E_n^0 - E_k^0}$

$\langle 1|\hat{H}_1|2\rangle = \langle 2|\hat{H}_1|1\rangle = -A$

$E_1^{(2)} = \frac{A^2}{2\mu\varepsilon}$; $E_2^{(2)} = -\frac{A^2}{2\mu\varepsilon}$