Physics 492: Recitation #5
February 16, 2017

1. Calculate the first order correction in energy to a particle in a one dimensional box $0 < x < a$ due to a step of height V_0 in the range $0 < x < a/2$. What does V_0 need to be small to for perturbation theory to be valid?

$$\int \sin^2(ny)dy = \frac{y}{2} - \frac{\sin(2ny)}{4n}$$

and for $\cos^2$ the same with a + sign.

2. Show that the matrix elements of perturbation theory satisfy,

$$\sum_m \left|(\hat{H}_1)_{nm}\right|^2 = (\hat{H}_1^2)_{nn}$$

3. Recall the Hamiltonian for the Ammonia molecule in the $|1\rangle$,$|2\rangle$ basis is

$$\left(\hat{H}\right)_{1,2} = \begin{pmatrix} E_0 + \mu\mathscr{E} & -A \\ -A & E_0 - \mu\mathscr{E} \end{pmatrix}$$

The exact solution is $E = E_0 \pm \sqrt{A^2 + (\mu\mathscr{E})^2}$. Assume $A << \mu\mathscr{E}$, and expand in a Taylor series keeping the leading term. Argue that the first order correction in perturbation theory should be zero. Show explicitly that the first order perturbation is zero. Find the second order correction and compare with the Taylor series expansion.