Coherent State Worksheet
Phys 521, Fall 2023

In class we showed that eigenstates of the harmonic oscillator creation operator (called coherent states)

$$\hat{a}\left|z\right\rangle = z\left|z\right\rangle$$

$$\left\langle z\right|\hat{a}^\dagger = z^*\left\langle z\right|$$

where z is a complex number $z = (y_0 + iq_0)/\sqrt{2}$.

We found that

$$\left|z\right\rangle = e^{-|z|^2/2}e^{z\hat{a}^\dagger}\left|0\right\rangle$$

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1. Find $\langle\hat{y}\rangle$, Δy and $\langle\hat{q}\rangle$, Δq for coherent states and show that coherent states have the minimum uncertainty product $\Delta x\Delta p$. Recall

$$\hat{y} = \hat{x}\sqrt{\frac{m\omega}{\hbar}};\quad \hat{q} = \hat{p}\frac{1}{\sqrt{m\omega\hbar}}$$

2. Show that the probability distribution $|\left\langle n\right\rangle z|^2$ is a Poisson with mean $|z|^2$.

3. Show that

$$\left|z\right\rangle = \exp\left(z\hat{a}^\dagger - z^*\hat{a}\right)\left|0\right\rangle$$

Hint: use the identity $e^{\hat{A}}e^{\hat{B}}e^{\frac{1}{2}[\hat{A},\hat{B}]} = e^{\hat{A}+\hat{B}}$ for $[\hat{A},\hat{B}]$ = a number (not an operator).

4. Show that $\exp\left(z\hat{a}^\dagger - z^*\hat{a}\right)$ is equal to a translation in momentum space times a translation in position space (up to an irrelevant phase factor).

5. show that

$$\langle E\rangle = \hbar\omega\left(|z|^2 + \frac{1}{2}\right)$$

and that

$$\Delta E = \hbar\omega|z|$$

and that therefore $\Delta E/\langle E\rangle \to 0$ as $|z| \to \infty$