

Discrete Symmetries① Parity  $\hat{\Pi}$  spatial inversionclassical  $\vec{r} \rightarrow -\vec{r}$ 

$$\hat{\Pi}^\dagger \hat{x} \hat{\Pi} = -\hat{x}$$

then  $\hat{x} \hat{\Pi} |x\rangle = -x(\hat{\Pi} |x\rangle)$

so  $\hat{\Pi} |x\rangle = e^{i\delta} |-x\rangle$  we can choose  $\delta$  by convention = 0

$$\hat{\Pi}^2 |x\rangle = |x\rangle \quad \text{so} \quad \hat{\Pi}^{-1} = \hat{\Pi}^\dagger = \hat{\Pi}$$

 $\hat{\Pi}$  is hermitian.Parity of momentum  $\hat{p}_x$ . Translation operator

$$T_x(a) |x\rangle = |x+a\rangle$$

$$\begin{aligned} T_x(-a) \hat{\Pi} |x\rangle &= T_x(-a) |-x\rangle = |-x-a\rangle = \hat{\Pi} |x+a\rangle \\ &= \hat{\Pi} \hat{T}(a) |x\rangle \end{aligned}$$

$$\text{or} \quad \hat{T}_x(-a) \hat{\Pi} = \hat{\Pi} \hat{T}(a)$$

infinitesimally,

$$\left(1 + i\varepsilon \frac{\hat{p}_x}{\hbar}\right) \hat{\Pi} = \hat{\Pi} \left(1 - i\varepsilon \frac{\hat{p}_x}{\hbar}\right)$$

$$\hat{p}_x \hat{\Pi} = -\hat{\Pi} \hat{p}_x$$

$$\pi P_x \pi = -P_x$$

in vector operator  $\vec{P} \xrightarrow{\pi} -\vec{P}$

orbital  $\vec{L} = \vec{r} \times \vec{p} \xrightarrow{\pi} \vec{L}$

$$\text{or } \pi \vec{L} \pi = \vec{L}$$

Similarly for spin,  $\pi \vec{S} \pi = \vec{S}$

If  $[\pi, H] = 0$  energy eigenstates are states of definite parity.

$$\pi |\psi_E\rangle = \pm |\psi_E\rangle$$

for  $|\psi_E\rangle = |n \ell m\rangle$

Parity is property of spherical harmonics!

$$\vec{r} \rightarrow -\vec{r} \quad \theta \rightarrow \pi - \theta, \quad \phi \rightarrow \phi + \pi$$

$$Y_\ell^m(\theta, \phi) \rightarrow (-1)^\ell Y_\ell^m(\theta, \phi)$$

$$\pi |n \ell m\rangle = (-1)^\ell |n \ell m\rangle$$

## Time reversal $\hat{T}$

Classically, position  $\vec{r} \rightarrow \vec{r}$   
momentum  $\vec{p} \rightarrow -\vec{p}$   
angular momentum  $\vec{J} \rightarrow -\vec{J}$

Em charge  $q \rightarrow q$ ; current  $\vec{j} \rightarrow -\vec{j}$

$$\vec{E} \rightarrow \vec{E} \quad ; \quad \vec{B} \rightarrow -\vec{B}$$

Schrödinger equation first order in time

$$i\hbar \frac{\partial}{\partial t} \psi = H \psi$$

If  $\psi(x, t)$  is solution,  $\psi(x, -t)$  is not  
due to  $\frac{\partial}{\partial t}$ . However  $\psi^*(x, -t)$  is solution.  
So  $\hat{T}$  includes complex conjugation.

$\hat{T}$  operator acting on arbitrary ket

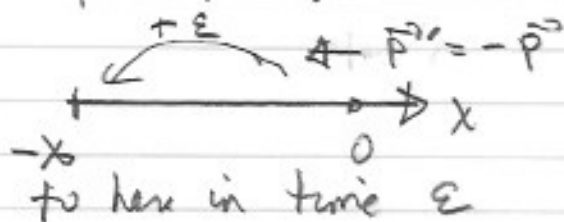
$$|\alpha\rangle \xrightarrow{\hat{T}} \hat{T}|\alpha\rangle$$

Consider time evolution over  
infinitesimal time  $\epsilon$ .

$$|\alpha, \epsilon\rangle = \left(1 - i\frac{\epsilon H}{\hbar}\right) |\alpha, 0\rangle$$

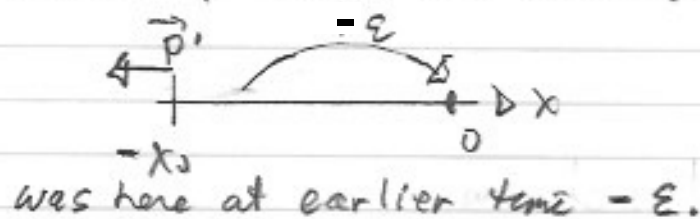
Consider two physically equivalent state evolutions. (Sakurai)

I. take  $\vec{p} \rightarrow \vec{p}' = -\vec{p}$  then evolves to  $x = -x_0$  in time  $\epsilon$



$$|\alpha'\rangle = \left(1 - \frac{i\hat{H}}{\hbar} \epsilon\right) \hat{\Pi} |\alpha\rangle$$

II. state  $\vec{p}'$  was at  $x = -x_0$  at time  $-\epsilon$



$$|\alpha'\rangle = \hat{\Pi} \left(1 + \frac{i\hat{H}}{\hbar} \epsilon\right) |\alpha\rangle$$

then  $-i\epsilon\hat{H}\hat{\Pi} = \hat{\Pi}i\epsilon\hat{H}$

We cannot have

$$i \hat{H} \hat{T} = \hat{T} (i \hat{H}) \not\rightarrow -\hat{H} \hat{T} = \hat{T} \hat{H}$$

because  $\hat{T}^{-1} \hat{H} \hat{T} = -\hat{H}$  would

give for free particles

$$\frac{\hat{p}^2}{2m} \xrightarrow{\hat{T}} -\frac{\hat{p}^2}{2m}$$

which is wrong.

$\hat{T}$  must be anti-unitary, equivalent to

$$\hat{T} = \hat{U} K \quad \hat{U}^\dagger = \hat{U}^{-1} \text{ unitary}$$

and complex conjugation operators

$$C \hat{K} = K C^* \quad \text{for any number } C$$

or, just use  $\hat{U} C = C^* \hat{U}$   
then

$$-i \hat{H} \hat{T} = \hat{T} (i \hat{H}) \Rightarrow \hat{H} \hat{T} = \hat{T} \hat{H}$$

$$\hat{T}^{-1} \hat{H} \hat{T} = \hat{H}$$

Preserves commutators,

$$[\hat{x}_i, \hat{p}_j] |\psi\rangle = i \hbar \delta_{ij} |\psi\rangle$$

$$\hat{T} [\hat{x}_i, \hat{p}_j] (\hat{T}^{-1} |\psi\rangle) = \overset{\text{anti-unitary}}{-i \hbar \delta_{ij}} (\hat{T} |\psi\rangle)$$

## Time reversal of spinor

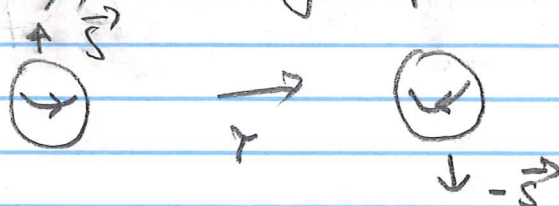
State  $\chi(\hat{n})$  is +1 eigenvalue

of  $\vec{\sigma} \cdot \hat{n}$  operator. On HW, you

$$\text{show } \hat{T} \chi(\hat{n}) = -i\sigma_2 \chi^*(\hat{n})$$

$$= \chi(-\hat{n}) \quad \text{flipped spin}$$

classically, rotating sphere



$$(-i\sigma_2)^2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = -\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = -I$$

$$\text{so } T^2 \chi(\hat{n}) = -\chi(\hat{n})$$

where as for spin 0 stat.  $T^2 |0\rangle = +|0\rangle$

generally,

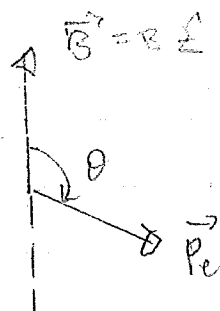
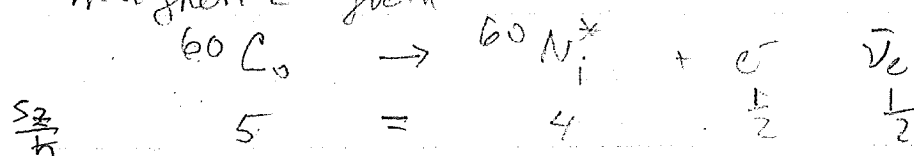
$$T^2 |\frac{1}{2} \text{ integer}\rangle = -|\frac{1}{2} \text{ integer}\rangle$$

$$T^2 |\text{integer}\rangle = +|\text{integer}\rangle$$

## Parity Violation in $\beta$ -decay

Wu et al. 1957

Cobalt source at 0.01 K polarized in magnetic field



Observed:  $\frac{dN_e}{d\Omega} = 1 - \frac{v}{c} \cos\theta$

since  $\vec{P}_e \xrightarrow{\pi} -\vec{P}_e$ , this preferred direction violates parity.

## Structure of weak interaction.

Weak decay produces left-handed "chiral" electron, spin anti-aligned with momentum.

we will discuss chirality next semester. For ultra-relativistic particle  $v \approx c$  chirality is equal to helicity.

# Electric Dipole Moment of neutron

neutron ( $u \frac{2}{3}, d -\frac{1}{3}, d -\frac{1}{3}$ ) quarks

$$\begin{array}{c} \frac{2}{3} \oplus \\ -\frac{1}{3} \ominus \end{array} \downarrow e = 0.1 r_n = 0.1 \text{ fm}$$

$$d_n = qe = 4 \times 10^{-4} \text{ e-cm}$$

experimentally  $d_n < 3 \times 10^{-26} \text{ e-cm}$

$\vec{d}$  proportional to neutron spin

$$\vec{d} = d \left( \frac{\vec{S}}{\frac{\hbar}{2}} \right)$$

under time reversal,  $\vec{S} \rightarrow -\vec{S}$  so  $\vec{d} \rightarrow -\vec{d}$   
electric field does not change:

$$\vec{E} \rightarrow \vec{E} \quad t \rightarrow -t$$

$$\vec{H}_d = \vec{d} \cdot \vec{E} \rightarrow -\vec{d} \cdot \vec{E} \quad \text{violation}$$

compare to magnetic moment:  $\vec{\mu} = \mu \frac{\vec{S}}{\frac{\hbar}{2}}$

but  $\vec{B} \rightarrow -\vec{B} \quad t \rightarrow -t$

$$\text{so } H_m = \vec{\mu} \cdot \vec{B} \rightarrow \vec{\mu} \cdot \vec{B}$$

also violates Parity or  $\vec{S} \xrightarrow{P} +\vec{S}$  axial vector  
 $\vec{d} \cdot \vec{E} \rightarrow -\vec{d} \cdot \vec{E}$ ;  $\vec{\mu} \cdot \vec{B} \rightarrow \vec{\mu} \cdot \vec{B}$   $\vec{E} \xrightarrow{P} -\vec{E}$  vector

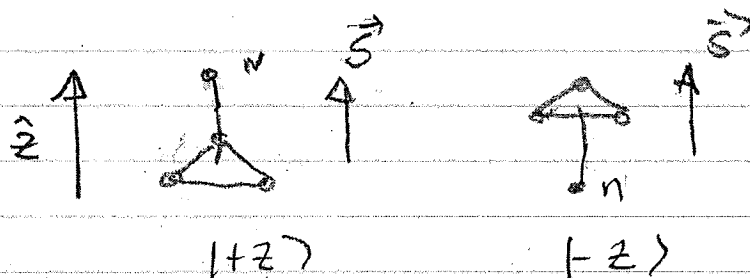


Note, many molecules have

large EDM's, for example

$$\text{NH}_3 \quad d = 0.3 \times 10^{-30} \text{ e.c.m}$$

but edm not proportional to spin.  
both states exist



note, ground state has zero electric dipole moment.

note QCD can explain neutron  
EDM,  $\theta$  parameter.

$\theta$  parameter "naturally"  $\sim 1$

but limit on neutron EDM  $\theta < 10^{-10}$

why so small?