HW #10 Problems
Quantum 521

1. Prove that the two-spin state $|0,0\rangle$ is invariant under rotations by an explicit change of basis. Consider a rotation about the $\hat{y}$ axis.

2. Find $\langle(\vec{\sigma}_2 \cdot \hat{b})(\vec{\sigma}_1 \cdot \hat{a})\rangle$ for the two-spin state $|0,0\rangle$ where $\hat{a} \cdot \hat{b} = \cos\theta$. Here $1, 2$ are particle labels, so the spin operator acts only on the corresponding particle spinor.

3. The annihilation of positronium in its ground state $^1S_0$ but *negative parity* produces two photons. The polarization of the $J = 0$ negative parity two-photon state is

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left[|RR\rangle - |LL\rangle\right]$$

   Show that this state has negative parity. (Under a parity transformation a vector changes sign, but a pseudo-vector such as angular momemtum $(\vec{r} \times \vec{p})$ does not.)

   What is the probability that photon 1 will be found to be x-polarized and photon 2 will be found to be y-polarized, that the system is in the state $|xy\rangle$? What is the probability that the system is in the state $|xx\rangle$ What these probabilities be if the state had positive parity?

4. Consider the matrix

$$U = \frac{a_o + i\vec{a} \cdot \vec{\sigma}}{a_o - i\vec{a} \cdot \vec{\sigma}}$$

   where a_o and a_i, $i = 1, 2, 3$ are all real.

   a) Prove that U is unitary and that det(U) = 1.

   b) In general, a 2x2 unitary matrix is equivalent to a rotation. Find the corresponing rotation matrix.