HW #11 Problems
Quantum 521

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1. a) For a plane wavefunction $\psi(x,t)$ show that $\psi^\star(x,-t)$ has the momentum reversed.

   b) Let the spinor with positive eigenvalue of $\sigma \cdot \hat{n}$ be denoted $\chi(\hat{n})$. Using the explicit form of $\chi(\hat{n})$ in terms of polar and azimuthal angles Euler angles, verify that the operator $-i\sigma_2\chi^\star(\hat{n}) = \chi(-\hat{n})$ flips the spin.

2. Write the spherical vector operater components V_q (rank 1 tensor) in terms of cartesian components V_x, V_y, V_z. Use the results of the HW3 for the rotation matrix for a spherical vector about the y-axis $d_{q,q'}$ to evaluate

   $$\sum_{q'} d_{q,q'} V_{q'}$$

   and show that this corresponds to the rotation of the cartesian components V_x, V_y, V_z about the y-axis.

3. In this problem from Shankar we will prove the projection theorem (15.3.19)

   Exercise 15.3.3. (1) Using $\langle jj|jj,10\rangle = [j/(j+1)]^{1/2}$ show that

   $$\langle \alpha j||J_1||\alpha' j'\rangle = \delta_{\alpha\alpha'}\delta_{jj'}\hbar[j(j+1)]^{1/2}$$

   (2) Using $\mathbf{J}\cdot\mathbf{A} = J_zA_z + \frac{1}{2}(J_-A_+ + J_+A_-)$ (where $A_\pm = A_x \pm iA_y$) argue that

   $$\langle \alpha' jm'|\mathbf{J}\cdot\mathbf{A}|\alpha jm\rangle = c\langle \alpha' j||A||\alpha j\rangle$$

   where c is a constant independent of α, α' and $\mathbf{A}$. Show that $c = \hbar[j(j+1)]^{1/2}\delta_{m,m'}$.

   (3) Using the above, show that

   $$\langle \alpha' jm'|A^q|\alpha jm\rangle = \frac{\langle \alpha' jm|\mathbf{J}\cdot\mathbf{A}|\alpha jm\rangle}{\hbar^2 j(j+1)}\langle jm'|J^q|jm\rangle \qquad (15.3.19)$$

4. a) Write the operator corresponding to products of the position vector components, xy, xz, $x^2 - y^2$ as components of an irreducible tensor operator of rank 2

   b) The quadrupole moment of a state $|\psi\rangle = |\alpha, j, m = j\rangle$ is

   $$Q = e\langle\psi|(3z^2 - r^2)|\psi\rangle$$

   Evaluate $\langle \alpha, j, m'|\,(x^2 - y^2)\,|\alpha, j, m = j\rangle$ in terms of Q and Clebsch-Gordon coeffiecients. Evaluate explicitly for $j = 1$.