HW #12 Problems
Quantum 521

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1. Find $\langle \hat{y} \rangle$, Δy and $\langle \hat{q} \rangle$, Δq for coherent states and show that coherent states have the minimum uncertainty product $\Delta x \Delta p$. Recall

$$\hat{y} = \hat{x}\sqrt{\frac{m\omega}{\hbar}}; \quad \hat{q} = \hat{p}\frac{1}{\sqrt{m\omega\hbar}}$$

2. Show that the probability distribution $|\langle n|z\rangle|^2$ is a Poisson with mean $|z|^2$.

3. Show that

$$|z\rangle = \exp\left(z\hat{a}^\dagger - z^*\hat{a}\right)|0\rangle$$

Hint: use the identity $e^{\hat{A}}e^{\hat{B}}e^{\frac{1}{2}[\hat{A},\hat{B}]} = e^{\hat{A}+\hat{B}}$ for $[\hat{A},\hat{B}]$ = a number (not an operator).

4. Show that $\exp\left(z\hat{a}^\dagger - z^*\hat{a}\right)$ is equal to a translation in momentum space times a translation in position space (up to an irrelevant phase factor).

5. show that

$$\langle E \rangle = \hbar\omega\left(|z|^2 + \frac{1}{2}\right)$$

and that

$$\Delta E = \hbar\omega|z|$$

and that therefore $\Delta E/\langle E \rangle \to 0$ as $|z| \to \infty$