HW #2 Problems
Quantum 521

1. Write a computer program (in, for example, Matlab or python) proving the central limit theorem. Start with a single, flat random number generated between 0 and 1.

2. A general procedure for generating random numbers distributed as the inverse transform method. Given that $f(x)$ is a probability density function (PDF), the cumulative probability $F(a) = \int_0^a f(x)dx$ is a uniform PDF. Choose a uniform random number $0 < u < 1$, then $x = F^{-1}$ will be a random number distributed according to $f(x)$. *(Think about the fundamental theorem of calculus.)* See Figure 1. This assumes that the function F can be inverted analytically. Use the inverse function theorem applied to Gaussian integrals to find how to generate pairs of Gaussian random numbers z_1, z_2 from pairs of uniform random numbers u_1, u_2 on the interval $(0, 1)$. Include your analytical solution for z_1, z_2. Write a Gaussian random number generator. Make a histogram showing that z_1, z_2 are Gaussian. Make a scatter plot proving that z_1, z_2 are uncorrelated.

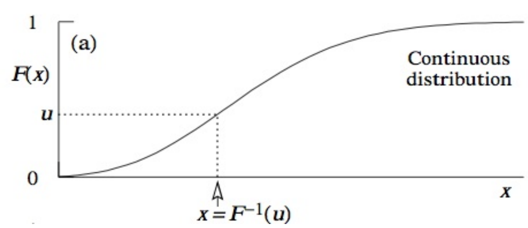

Figure 1: illustration of inverse transform method (from the Particle Data Group).

3. (*from R. Baierlein*) A lawn sprinkler is made from a spherical cap (max angle $\theta = 45°$) with a large number of identical holes, with density $n(\theta)$. Determine $n(\theta)$ such that the water is uniformly sprinkled over a circular area. The surface of the cap is level with the lawn. Assume that the size of the cap is negligible compared to the size of the lawn to be watered and neglect air resistance. Sketch your answer for $n(\theta)$.

4. Starting with the Hamiltonian for a free relativistic particle, obtain the corresponding Lagrangian from the Legendre transformation. Show that it gives the correct non-relativistic limit.

Exercise 2.8.7. Consider the harmonic oscillator, for which the general solution is

$$x(t) = A \cos \omega t + B \sin \omega t.$$

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Express the energy in terms of A and B and note that it does not depend on time. Now choose A and B such that $x(0) = x_1$ and $x(T) = x_2$. Write down the energy in terms of x_1, x_2, and T. Show that the action for the trajectory connecting x_1 and x_2 is

$$S_{\text{cl}}(x_1, x_2, T) = \frac{m\omega}{2 \sin \omega T}[(x_1^2 + x_2^2) \cos \omega T - 2x_1 x_2].$$

Verify that $\partial S_{\text{cl}}/\partial T = -E$.

Figure 2: Shankar 2.8.7