HW #4 Problems
Quantum 521

---

1. Use uncertainty principle to estimate ground state for the simple harmonic oscillator.

2. For a particle in a box with side L on the interval $= -L/2 < x < L/2$ and in the state $\psi = A(L/2 + x)(L/2 - x)$ calculate the probability to measure the ground state energy. (A is a normalization constant.) Get a numerical value!

3. (this one is long) Time evolution of Gaussian Ψ another way. Use the time evolution of a free partice $\hat{U}(t) = exp(-it\hat{H}/\hbar)$. Fourier transform the Gaussian wave function. Apply the time evolution operator. Fourier transform back. What is the time evolution of the width of the probability density and what is the characteristic time τ of the spreading? Calculate the time evolution of the uncertainty product in terms of τ.

4. For an exponentially decaying state with lifetime τ, derive a limit using the time-energy uncertainty relation on the time t_0 required to measure the exponential decay,

$$t_0 > \tau \ln \left[1 + \left(\frac{\hbar}{2\tau \Delta E} \right)^2 \right]$$

where ΔE is the uncertainty on the measurement of the state energy. Refer to example in lecture 6. How does this limit effect the measurement of laboratory lifetimes such as alpha or beta decay?