HW #5 Problems
Quantum 521

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1. For the density matrix for a single-particle spin-1/2 state with polarization $\hat{p}$ written as

$$\rho = \frac{1}{2}\left[I + \hat{p}\cdot\vec{\sigma}\right]$$

prove that $\langle\vec{S}\rangle = \frac{\hbar}{2}\hat{p}$

2. a) For a mixed ensemble of spin 1/2 systems, knowing the all three components of the polarization $\langle S_i\rangle$ write the density matrix.

b) Now consider a pure ensemble of spin 1/2 systems. Suppose you know $\langle S_x\rangle, \langle S_z\rangle$ but only the sign of $\langle S_y\rangle$. Show that this completely determines the state vector.

3. Suppose we have a pure ensemble at $t = 0$. Show that it cannot evolve into a mixed ensemble.

4. (From Commins) For an electron in a magnetic field, use the density matrix to formalism to calculate the time evolution of the expectation value of the spin and show that it corresponds to classical spin precession.