HW #6 Problems
Quantum 521

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1. A polished silicon surface can act as an impenetrable barrier for neutrons. Imagine that a neutron is "placed" (negligible kinetic energy) above such a mirror. Estimate the height (in microns) that the neutron would float above the mirror. For this estimate use $m_N c^2 = 1000$ MeV, $\hbar c = 200$ eV-nm, and $m_N g = 10^{-13}$eV/μm where g is the acceleration of gravity at the earth's surface.

   Note that the exact solution is found in terms of the Airy function. See Commins, p. 116.

2. Just as for the momentum operator in the position basis, we can work out the position operator in the momentum basis. Define the operator $\mathrm{\hat{W} = exp\{ip_0\hat{x}/\hbar\}}$, and work out $\langle p| \hat{x} |p'\rangle$. Check that in this basis we get $[\hat{x}, \hat{p}] = i\hbar$.

3. For the delta-function potential given in class, show that the S-matrix has a simple pole on the imaginary axis. Prove that this pole corresponds to the value of the bound state for the delta-function potential. Solve the delta-function potential bound state problem to prove this. Plot the transmission coefficient T as a function of $x = kb$ with x real.

4. Calculate the transmission coefficient for a particle scattering off of a a rectangular barrier of height V_0 and width a with energy $E > V_0$. Analytically continue the result to get the tunneling transmission coefficient.