HW # 8 Solutions

1)

$$V = -\vec{\mu}\cdot\vec{B} \qquad \mu = \left(\frac{g}{2}\right)\left(\frac{e\hbar}{2mc}\right)$$

difference in action between two paths,

$$\Delta S = \int L\,dt = \int_{\ell} \mu B \frac{dx}{v} = \frac{\mu m_N B}{\ell v}$$

$$= \frac{\mu m_N B}{2\pi\hbar}\ell\lambda \quad \text{with } p = \frac{2\pi\hbar}{\lambda}$$

condition for successive maxima:

$$\frac{\Delta S}{\hbar} = 2\pi = \frac{1}{2\pi\hbar^2}\mu m_N B\ell\lambda$$

$$\Delta B = \frac{4\pi^2\hbar^2}{\mu m_N \ell\lambda} \qquad \mu m_N = \frac{ge\hbar}{4c}$$

$$= \frac{4\pi^2\hbar^2}{\frac{ge\hbar}{4c}(\ell\lambda)} = \frac{16\pi^2\hbar c}{e\ell\lambda}$$

2) For free particle, $c = 0$

$$\frac{d^2 f}{dt^2} = 0 \Rightarrow f = t$$

$$A(t) = \sqrt{\frac{m}{2\pi i \hbar t}}$$

$$\mathcal{L} = \frac{m\dot{x}^2}{2} \; ; \; S = \int_{t_1}^{t_2} \frac{m\dot{x}^2}{2} dt = \frac{m\dot{x}^2}{2}(t_2 - t_1)$$

$\dot{x} = \text{constant}$

free particle propagator is then

$$\langle x_2 | U(t) | x_1 \rangle = \sqrt{\frac{m}{2\pi i \hbar t}} \, e^{\frac{i}{\hbar}\frac{m\dot{x}^2 t}{2}}$$

where $\dot{x} = \frac{x_2 - x_1}{t}$

3) $m\frac{d^2 f}{dt^2} + \frac{1}{2} m\omega^2 f = 0 \qquad f(0) = 0, \; \left.\frac{df}{dt}\right|_0 = 1$

note c = mω^2

$$f = \frac{\sin \omega t}{\omega} \qquad A(t) = \sqrt{\frac{m\omega}{2\pi i \hbar \sin \omega t}}$$

$$\langle x_b | \hat{U}(t) | x_a \rangle = \sum \langle x_b | n \rangle \langle n | x_a \rangle e^{-i E_n t/\hbar}$$

let $t \to -i\tau$ $\qquad$ limit $\tau \to \infty$

$$\langle x_b | \hat{U}(\tau) | x_a \rangle \longrightarrow \langle x_b | 0 \rangle \langle x_a | 0 \rangle e^{-\tau E_0/\hbar}$$

$$S[\bar{x}] = \frac{m\omega}{2\sin\omega t}\left[\cos\omega t\,(x_b^2 - x_a^2) - 2x_a x_b\right]$$

$t \to -i\omega\tau$

$$\sin(-i\omega\tau) = \frac{1}{i}\sinh(\omega\tau)$$
$$\cosh(-i\omega\tau) = \cosh(\omega\tau)$$

$$S[\bar{x}] \to \frac{im\omega}{2\sinh\omega\tau}\left[\cosh\omega\tau\,(x_a^2 + x_b^2) - 2x_a x_b\right]$$

in limit $\tau \to \infty$ $\quad \sinh(\omega\tau) \to \frac{1}{2}e^{\omega\tau}$

$\qquad\qquad\qquad\quad \cosh(\omega\tau) \to \frac{1}{2}e^{\omega\tau}$

$$\frac{iS[\bar{x}]}{\hbar} \xrightarrow[\tau\to\infty]{} \frac{-m\omega}{2\hbar}\, 2e^{-\omega\tau}\left[\frac{e^{\omega\tau}}{2}(x_a^2 + x_b^2)\right]$$

$$= \frac{-m\omega}{2\hbar}(x_a^2 + x_b^2)$$

$$A(\tau) = \left[\frac{m\omega}{2\pi\hbar\sinh(\omega\tau)}\right]^{1/2} \xrightarrow[\tau\to\infty]{} \sqrt{\frac{m\omega}{\hbar}}\, e^{-\omega\tau/2}$$

then comparing these 2 results

$$\langle x_b|0\rangle\langle x_a|0\rangle e^{-\tau E_0/\hbar} = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-\frac{m\omega x_b^2}{2\hbar}}$$
$$\times \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-\frac{m\omega x_a^2}{2\hbar}}\, e^{-\tau\omega/2}$$

giving

$$\langle x|0\rangle = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-\frac{m\omega x^2}{2\hbar}}$$

and $\quad E = \hbar\omega/2$