HW #9 Problems
Quantum 521

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1. Prove that the product of an infinite number of infinitessimal transformations is an exponential:

$$\lim_{n \to \infty} \left(1 - \frac{i\theta_i \hat{T}_i}{n}\right)^n = \exp\left(-\mathrm{i}\theta_\mathrm{i}\hat{\mathrm{T}}_\mathrm{i}\right)$$

2. prove $[J_i J_i, J_j] = 0$

3. Find the three explicit matricies for the SU(2) generators of the 3-dimensional representation.

4. Derive Rabi's magnetic resonance formula.

5. Consider a spin-1 particle with magnetic moment q and g-factor g in a static, uniform magnetic field taken to be $\vec{B} = B\hat{z}$. At $t = 0$ the particle is in the eigenstate of $\hat{S}_y$ with eigenvalue $+\hbar$. Find $|\psi(t)\rangle$ and $\langle \hat{S}_y(t)\rangle$. Calculate $\langle \hat{S}_y(t)\rangle$ from $\langle [H, S_y]\rangle$ and show the result is consistent.

6. The expectation values of the operators J are Euclidean vectors. For $j = 1/2$, using the commutation relations for the spin operators, prove explicity that the expectation values of S_y transform as elements of a Euclidean vector for rotations about the $\hat{z}$ axis.

7. The expectation value of the three spin operators are components of a Euclidean vector. Suppose an electron is in the state with $\langle s_z\rangle = +\hbar/2$, that is the state $|+z\rangle$. What is $\langle s'_z\rangle$ where the z' makes an angle θ with respect to the z axis? With respect to the z' axis eigenstates $|\pm z'\rangle$, the electron state can be written as (this is just another basis)

$$|+z\rangle = \alpha\,|+z'\rangle + \beta\,|-z'\rangle$$

   Calculate $\langle s'_z\rangle$ in terms of the probabilities α^2 and β^2 and determine these probabilities in terms of the angle θ.