Phys 521

Fall 2023

Lec #12 EPR

Einstein, Podolsky, Rosen (1935)

① Entangled states

multiparticle states are linear combinations of direct products:

$$|\alpha\rangle_1 \otimes |\beta\rangle_2$$

α, β state label
1, 2 particle label

This simple two-particle state is separable. Linear combinations are not necessarily separable. Not separable ≡ entangled

Consider spin state of two electrons combined into state of total spin 0:

$$|0,0\rangle = \frac{1}{\sqrt{2}}\left(|+z\rangle_1 |-z\rangle_2 - |-z\rangle_1 |+z\rangle_2\right)$$

Particles in an entangled state can exhibit non-local correlations (non-locality)

"There is a troubling weirdness about quantum mechanics." - S. Weinberg

We will consider this entangled two-spin state which is a very rare decay of the neutral pion

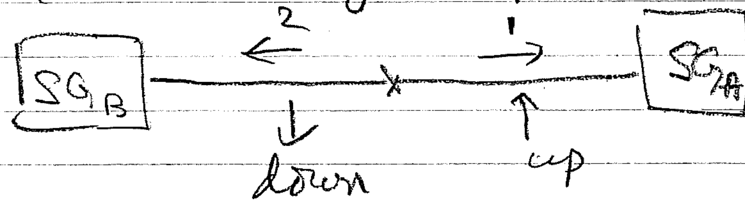
$$B_r(\pi^0 \to \gamma\gamma) = 98.8\%$$

$$B_r(\pi^0 \to e^+e^-) = 6.46 \times 10^{-8}$$

(Sakurai considers similar rare decay $\eta \to \mu^+\mu^-$)

π^0 has spin 0. To conserve angular momentum, e^+e^- spin state is entangled. Similar entangled states can be produced of two photons.

Consider abstract example of two particles produced in $|0,0\rangle$ state. Known as Bell-type experiment.



If SG measure same direction, only possible outcomes are:

1	2	Prob.
+	−	1/2
−	+	1/2
+	+	0
−	−	0

This correlation happens across any spatial separation of measurements.

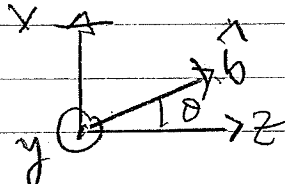
Suppose A is done first, and measures spin up + for particle 1. How does particle 2 "know" to be in state down?

More generally, consider two arbitrary directions.

You will show, $\langle 0,0|\sigma_b^2\sigma_a^1|0,0\rangle = -\hat{a}\cdot\hat{b}$

What about Bell probabilities?

Without loss of generality, take $\hat{a} = \hat{z}$ and $\hat{b}$ in the $\hat{x}$-$\hat{z}$ plane,



Basis states related by Rotation,

$$(|+b\rangle, |-b\rangle) = (|+a\rangle, |-a\rangle)\begin{pmatrix} c & -s \\ s & +c \end{pmatrix}$$

where $c \equiv \cos\theta/2$, $s \equiv \sin\theta/2$

or
$$|+b\rangle = c\,|+a\rangle + s\,|-a\rangle$$
$$|-b\rangle = -s\,|+a\rangle + c\,|-a\rangle$$

amplitudes are;

$$A(+a,+b) = \langle +a,+b|0,0\rangle = \frac{1}{\sqrt{2}}\,{}_2\langle +b|-a\rangle_2$$
$$A(+a,-b) = \langle +a,-b|0,0\rangle = \frac{1}{\sqrt{2}}\,{}_2\langle -b|-a\rangle_2$$

etc.

All four possibilities are

outcome 1,2	Amplitude $_2\langle \,|\, \rangle_2$	P(1,2)
$+a, +b$	$\frac{1}{\sqrt{2}}\langle +b \| -a\rangle$	$= \frac{1}{2}\sin^2\theta/2$
$+a, -b$	$\frac{1}{\sqrt{2}}\langle -b \| -a\rangle$	$= \frac{1}{2}\cos^2\theta/2$
$-a, +b$	$-\frac{1}{\sqrt{2}}\langle +b \| +a\rangle$	$= \frac{1}{2}\cos^2\theta/2$
$-a, -b$	$-\frac{1}{\sqrt{2}}\langle -b \| +a\rangle$	$= \frac{1}{2}\sin^2\theta/2$

note $\sum P_i = 1$ and another way to get

$$\langle \sigma_b^2 \sigma_a^1 \rangle = P(+a,+b) + P(-a,-b) - P(+a,-b) - P(-a,+b)$$

$$= \sin^2\theta/2 - \cos^2\theta/2 = -\cos\theta$$

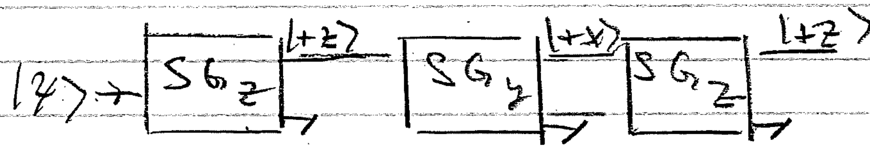
You can calculate this directly all in the z-basis

EPR- Hidden variable theory

They argue that these correlations arise from local quantities that remain hidden until measured. "They" also called <u>local realists</u>. To be consistent with non-commuting observables (e.g. $[S_x, S_y] \neq 0$) must assume that only one incompatible observable can be measured at a time. Write local realist "state" as

$$\{+z, +x\} \quad \text{etc.}$$

Consider sequence of S.G. measurements of spin. In Q.M.,



$$\text{prob } P_T = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$$

Each measurement causes <u>collapse</u> of wave function to spin eigenstate with probability $1/2$

Local realist description

$$\{\psi\} = \frac{1}{4}\{+z,+x\} + \frac{1}{4}\{+z,-x\} + \frac{1}{4}\{-z,+x\} + \frac{1}{4}\{-z,-x\}$$

$$SG_z \rightarrow \frac{1}{4}\{+z,+x\} + \frac{1}{4}\{+z,-x\} \qquad P = \frac{1}{2}$$

$$SG_x \rightarrow \frac{1}{2}\left[\frac{1}{4}\{+z,+x\} + \frac{1}{4}\{-z,+x\}\right]$$

hidden variable gets "shaken"

$$SG_z \rightarrow \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{4}\left[\{+z,+x\} + \{+z,-x\}\right]$$

shaken

$$P_{TOT} = \frac{1}{8}$$

Local realist must assume measurement "shakes" hidden variable with equal probabilities.

Local realist considers this uncontrolled disturbance of hidden variable to be preferrable to quantum non-locality

Local Realist description of Bell-type experiment

states are all 4 combinations of $\hat{a}\cdot\hat{b} = \cos\theta$

$$\{\pm a, \pm b\}_1 + \{\pm a, \pm b\}_2$$

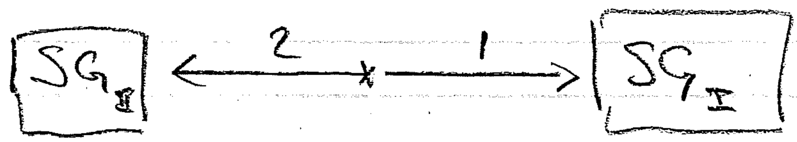
outcome 1,2	hidden variable state		Q.M. P(1,2)
$+a, +b$	$\{+a, -b\}_1$	$\{-a, +b\}_2$	$\frac{1}{2}\sin^2\theta/_2$
$+a, -b$	$\{+a, +b\}_1$	$\{-a, -b\}_2$	$\frac{1}{2}\cos^2\theta/_2$
$-a, +b$	$\{-a, -b\}_1$	$\{+a, +b\}_2$	$\frac{1}{2}\cos^2\theta/_2$
$-a, -b$	$\{-a, +b\}_1$	$\{+a, -b\}_2$	$\frac{1}{2}\sin^2\theta/_2$

maintaining perfect anti-correlation of spins. Somehow, the probabilities for each arrangement in the ensemble must be set in advance to correspond to Q.M. result.

By "magic", classical probabilities are set in advance for any choice of θ!

Simple (Sakurai)

Bell Inequality example



SG I, II can be set to any of three directions $\vec{a}, \hat{b}, \hat{c}$.

Hidden variable stat, each particle 1,2 must have values prior to measurement.

Number	particle 1	particle 2	
N_1	$+a+b+c$	$-a-b-c$	$P(+a_1,+c_2)$
N_2	$+a+b-c$	$-a-b+c$	
N_3	$+a-b+c$	$-a+b-c$	
N_4	$+a-b-c$	$-a+b+c$	$P(+c_1,+b_2)$
N_5	$-a+b+c$	$+a-b-c$	
N_6	$-a+b-c$	$+a-b+c$	
N_7	$-a-b+c$	$+a+b-c$	
N_8	$-a-b-c$	$+a+b+c$	

$P(+a_1,+b_2)$ [N_3, N_4]

Where N_i are chosen to fit experiment.

However, must always have

$$P(+a_1,+b_2) \le P(+a_1,+c_2) + P(+c_1,+b_2)$$

since

$$N_3 + N_4 \le (N_2 + N_4) + (N_3 + N_7)$$

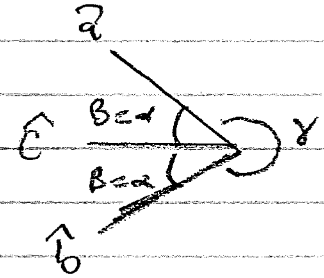
Q.M. prediction is:

$$\frac{1}{2}\sin^2\left(\frac{\gamma}{2}\right) \overset{?}{\leq} \frac{1}{2}\sin^2\left(\frac{\beta}{2}\right) + \frac{1}{2}\sin^2\left(\frac{\alpha}{2}\right)$$

Which will violate inequality if, for example,

$\gamma = 2\pi - \alpha - \beta$ (w/ all three directions in a plane)

and $\alpha = \beta = \pi/3$



$$\sin^2(120°) \leq 2\sin^2(30°)$$

$$\left(\frac{\sqrt{3}}{2}\right)^2 \nleq 2\left(\frac{1}{2}\right)^2$$

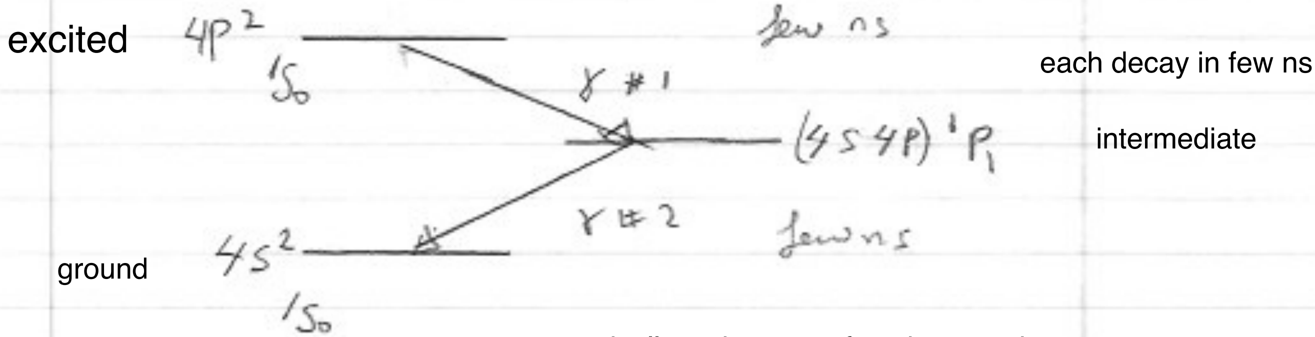
$$3/4 > \frac{1}{2}$$

Measurement violates Bell inequality if Quantum Mechanics is correct theory

EPR with Photons   Commins 3.3

Calcium $[1s^2 2s^2 2p^6 3s^2 3p^6]\, 4s^2$ ground state

atoms prepared in excited state

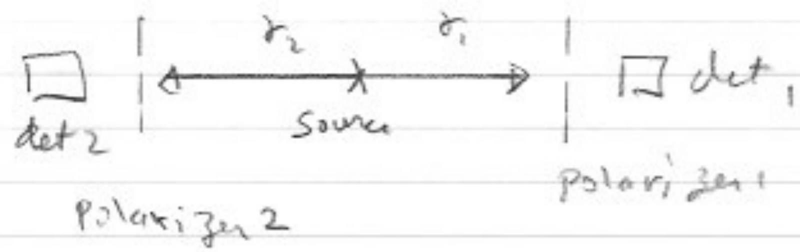


optically active wave function notation

2 optically active electrons,   ``term symbol": $^{2S+1}L_J$

$^1S_0$ $s=0,\ \ell=0,\ j=0$ spin of electron
$^1P_1$ $s=0,\ \ell=1,\ j=1$ does not change

$\gamma_1 \gamma_2$ almost simultaneous



Conservation of angular momentum and parity requires photons to be in entangled state.

$|++\rangle$ helicity +   helicity +
$\uparrow\uparrow$
1 2 $\Leftarrow$ $\Rightarrow$ $\xrightarrow{}\ \hat{z}$ along $\vec{p}$

$\longleftarrow$ p_2 $\longrightarrow$ p_1

helicity: spin along momentum direction

$|--\rangle$ $\Rightarrow$ $\Leftarrow$ $\longrightarrow\ \hat{z}$ opposite $\vec{p}$

$\longleftarrow$ p_2 $\longrightarrow$ p_1

Parity eigenstate. State produced w/ positive parity ($\vec{r} \to -\vec{r}$ spatial inversion)

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|++\rangle + |--\rangle \right)$$

in terms of linear polarization

$$|\pm\rangle_1 = \frac{1}{\sqrt{2}}\left(|\underline{x}\rangle \pm i|\underline{y}\rangle \right) \qquad \vec{p} = p\hat{z}$$

$$|\pm\rangle_2 = \frac{1}{\sqrt{2}}\left(|\underline{x}\rangle \mp i|\underline{y}\rangle \right) \qquad \vec{p} = -p\hat{z}$$

$-\vec{z}$

substitution gives

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\underline{x}\,\underline{x}\rangle + |\underline{y}\,\underline{y}\rangle \right)$$

entangled linear polarization

Polarizer projection operators:

$$|x'\rangle_1 \langle x'|_1 \qquad |x\rangle_2 \langle x|_2$$

drop underbar

Coincidence rate measuring x' on γ_1 & x on γ_2

$$|\psi\rangle = \frac{1}{\sqrt{2}} |x\rangle_2 \langle x|_2 \left[|x'\rangle\langle x'|x\,x\rangle + |x'\rangle\langle x'|y\,y\rangle \right]$$

$$= \frac{1}{\sqrt{2}} |x\rangle_2 \langle x|_2 \left[\cos\theta\ |x'x\rangle + \sin\theta\ |x'y\rangle \right]$$

$$\hat{x}' \cdot \hat{x} \equiv \cos\theta$$

$$= \frac{1}{\sqrt{2}} \cos\theta\ |x'x\rangle$$

Thus $P(\hat{x}'; \hat{x}) = \frac{1}{2}\cos^2\theta$ observed in experiment

In experiments Aspect et al.	PRL 47 (460) 1981 PRL 49 (1804) 1982

polarizers 1, 2 switched synchronously ($\Delta t = 5\,\text{ns} < \frac{L}{c}$)
between

$\hat{a}, \hat{a}'$ polarizer 1

$\hat{b}, \hat{b}'$ polarizer 2

4 possible configurations

measured by PMTs separated by $L = 6.5\,\text{m}$
with $L/c \cong 22\,\text{ns}$.

Generalized CHSH Bell inequality:
in terms of normalized coincidence rates:

$$S = R(\hat{a},\hat{b}) - R(\hat{a},\hat{b}') + R(\hat{a}',\hat{b})$$
$$+ R(\hat{a}',\hat{b}') - R(\hat{a}',-) - R(-,\hat{b})$$

"–" means polarizer removed

Bell inequality $\quad -1 \le S \le 0$

$S_{exp} = 0.126 \pm 0.014$ PRL 1981

$S_{QM\ theory} = 0.188 \pm 0.005$

and in 1982 $\quad -2 \le S' \le 2 \quad$ different Bell S'

$S'_{exp} = 2.697 \pm 0.015$

$S'_{theory} = 2.70 \pm 0.05$

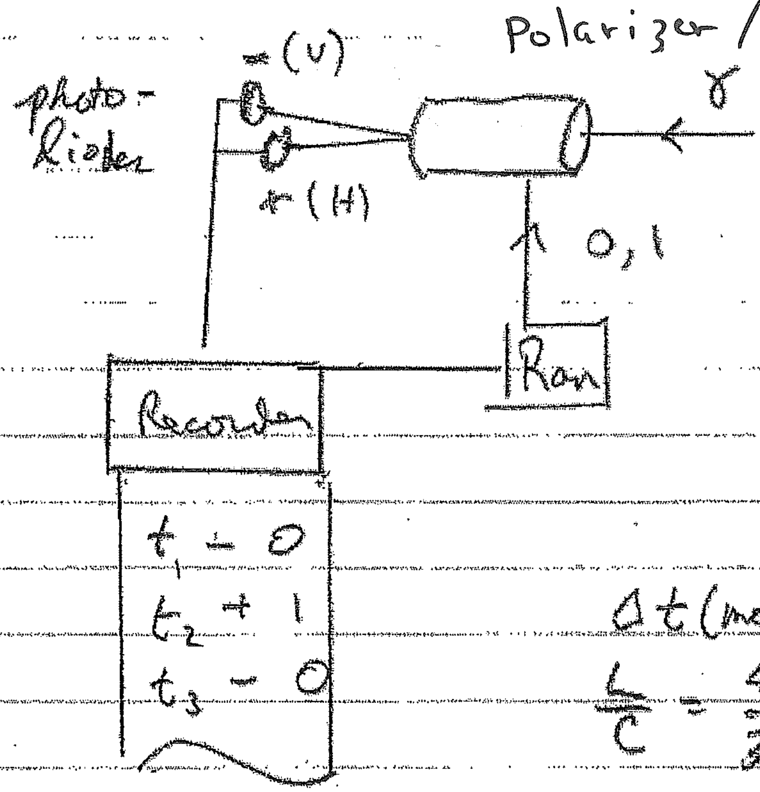
Weihs et al., PRL 81 (5039) 1998

Loopholes ① inefficient detectors
② spacelike separation of "observers"
(not sinusoidal switching like Aspect)

Source – "degenerate type-II parametric down conversion"

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|H\,V\rangle - |V\,H\rangle\right)$$

observers (Alice, Bob)
~~simplified~~ modulated
Polarizer / beam splitter



1 of 2 observers

0: θ
1: $\theta + 45°$
$\theta = \alpha, \beta$
for 2 observers

$\Delta t(\text{measure}) < 0.1\,\mu s$

$$\frac{L}{c} = \frac{400\,m}{300\,m/\mu s} = 1.3\,\mu s$$

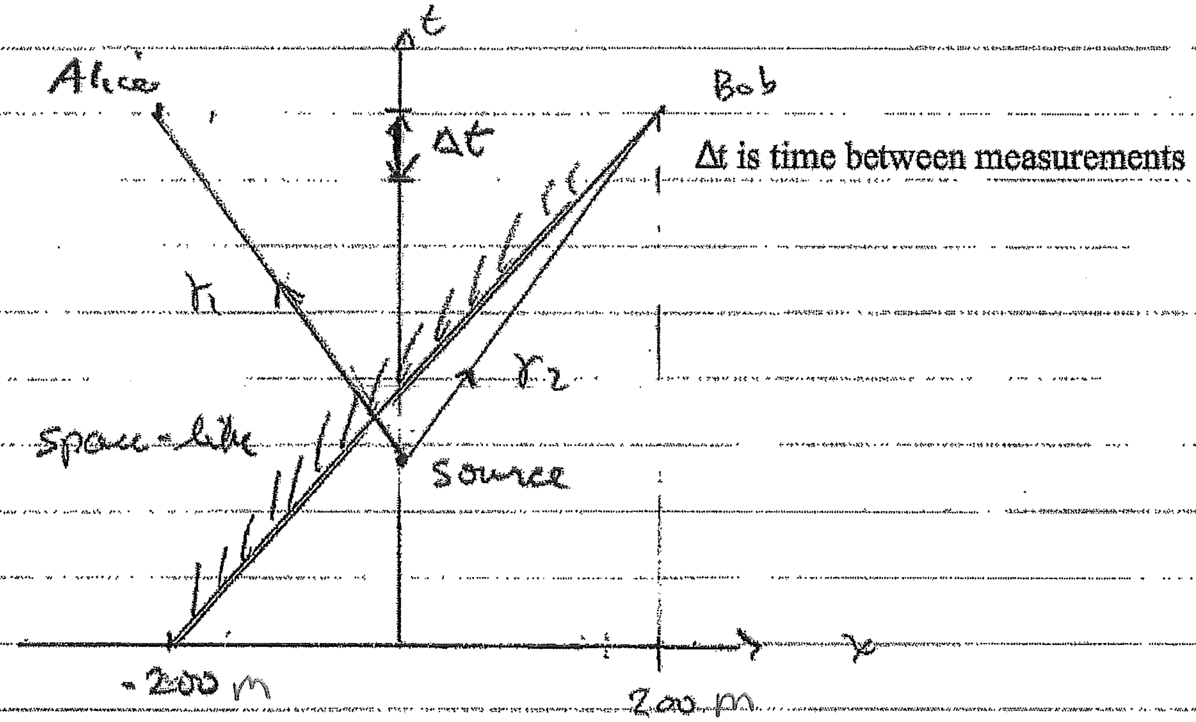
Expectation value E is

$$E(\alpha,\beta) \equiv \frac{1}{N}\left(C_{++} + C_{--} - C_{+-} - C_{-+}\right) \quad C \equiv \text{count}$$

$C_{++}^{QM} \propto \sin^2(\beta-\alpha)$ QM predicted counts

$$E^{QM}(\alpha,\beta) = -\cos(2(\beta-\alpha))$$

space-time diagram



light fiber 250 m length

Generalized Bell inequality –

$$S(\alpha,\alpha',\beta,\beta') = |E(\alpha,\beta) - E(\alpha',\beta)| + |E(\alpha,\beta') + E(\alpha',\beta')| \leq 2$$

$$S^{QM}_{max}(0, 45^\circ, 22.5^\circ, 67.5^\circ) = 2\sqrt{2} = 2.82$$

due to imperfect correlation "visibility" of source (97%) expect $S \simeq 2.74$

$$S^{exp.} = 2.73 \pm 0.02$$

"Expecting that any improved experiment will also agree with quantum theory, a shift of our classical philosophical positions seems necessary." Hensen et al.

$$N = 14700$$

"Loophole-free Bell inequality violation using electron spins separated by 1.3 kilometres", Hensen et al., 682 I NATURE I VOL 526 I 29 OCTOBER 2015