Phys 521

Recitation 1 Solutions

① $\hat{A}$ is diagonalized by similarity transformation,

$$S^\dagger A S = \mathrm{diag}(a_1, \dots a_n)$$

cyclic property of trace,

$$\mathrm{Tr}(S^\dagger A S) = \mathrm{Tr}(A S S^\dagger) = \mathrm{Tr}(A) = \sum a_i$$

② $\vec{L} = \bar{\bar{I}} \cdot \vec{\omega}$

moment of inertia tensor

$$I_{ij} = \sum_\alpha m_\alpha \left[r_\alpha^2 \delta_{ij} - (r_\alpha)_i (r_\alpha)_j \right]$$

can be diagonalized with three principle axes $\vec{\omega}_i$ and eigenvalue I_{ii}. Along their axis,

usual procedure

$$\vec{L} = I_{ii} \vec{\omega}_i$$

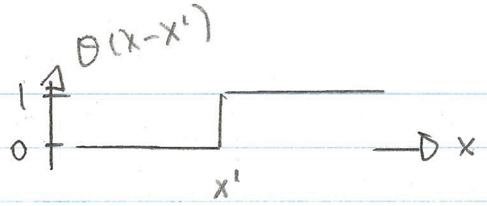
③ $$f(x) = \underbrace{f(x_0)}_{=0} + \underbrace{\left.\frac{df}{dx}\right|_{x_0}}_{\text{constant } a} (x - x_0) + \cdots$$

Then near x_0

$$\delta(f(x)) = \delta(a(x - x_0)) = \frac{1}{|a|} \delta(x_0 - x)$$

δ-function is symmetric, $\delta(x - x_0) = \delta(x_0 - x)$

4)



The derivative of θ is zero everywhere except for $x = x'$. Consider a small neighborhood about x', $x = x' + \varepsilon$, $x' = x - \varepsilon$:

$$\int_{-\infty}^{\infty} \frac{d\theta}{dx} f(x)\, dx = \lim_{\varepsilon \to 0} \int_{x'-\varepsilon}^{x'+\varepsilon} \frac{d\theta}{dx} f(x)\, dx$$

$$= f(x') \lim_{\epsilon \to 0} \int \underbrace{\frac{d\theta}{dx}\, dx}_{\text{total derivative}} = f(x') \lim_{\epsilon \to 0} \theta \Big|_{x'-\varepsilon}^{x'+\varepsilon}$$

$$= f(x')$$

$\frac{d\theta}{dx}$ behaves under integration just like $\delta(x - x')$.