Phys 521 Fall 2023

Recitation #10 Solutions

① reading the table, $\frac{1}{2} \times \frac{1}{2}$

$$|1,1\rangle = |\tfrac{1}{2},\tfrac{1}{2}\rangle_1 \otimes |\tfrac{1}{2}\,\tfrac{1}{2}\rangle_2$$

$$|1,0\rangle = \frac{1}{\sqrt{2}}\left(|\tfrac{1}{2},\tfrac{1}{2}\rangle_1 |\tfrac{1}{2},-\tfrac{1}{2}\rangle_2 + |\tfrac{1}{2}-\tfrac{1}{2}\rangle_1 |\tfrac{1}{2}\,\tfrac{1}{2}\rangle_2\right)$$

$$|1,-1\rangle = |\tfrac{1}{2},-\tfrac{1}{2}\rangle_1 \otimes |\tfrac{1}{2},-\tfrac{1}{2}\rangle_2$$

$$|0,0\rangle = \frac{1}{\sqrt{2}}\left(|\tfrac{1}{2}\,\tfrac{1}{2}\rangle_1 |\tfrac{1}{2}-\tfrac{1}{2}\rangle_2 - |\tfrac{1}{2}-\tfrac{1}{2}\rangle_1 |\tfrac{1}{2}\,\tfrac{1}{2}\rangle_2\right)$$

$1 \times \frac{1}{2}$

$$|\tfrac{3}{2},\tfrac{3}{2}\rangle = |1\,1\rangle |\tfrac{1}{2}\,\tfrac{1}{2}\rangle$$

$$|\tfrac{3}{2}\,\tfrac{1}{2}\rangle = \sqrt{\tfrac{1}{3}}\,|1,1\rangle |\tfrac{1}{2}-\tfrac{1}{2}\rangle + \sqrt{\tfrac{2}{3}}\,|1,0\rangle |\tfrac{1}{2}\,\tfrac{1}{2}\rangle$$

$$|\tfrac{3}{2}-\tfrac{1}{2}\rangle = \sqrt{\tfrac{2}{3}}\,|1,0\rangle |\tfrac{1}{2}-\tfrac{1}{2}\rangle + \sqrt{\tfrac{1}{3}}\,|1,-1\rangle |\tfrac{1}{2}+\tfrac{1}{2}\rangle$$

$$|\tfrac{3}{2},-\tfrac{3}{2}\rangle = |1,-\tfrac{1}{2}\rangle |\tfrac{1}{2}-\tfrac{1}{2}\rangle$$

$$|\tfrac{1}{2}\,\tfrac{1}{2}\rangle = \sqrt{\tfrac{2}{3}}\,|1,1\rangle |\tfrac{1}{2}-\tfrac{1}{2}\rangle - \sqrt{\tfrac{1}{3}}\,|1,0\rangle |\tfrac{1}{2}\,\tfrac{1}{2}\rangle$$

$$|\tfrac{1}{2}-\tfrac{1}{2}\rangle = \sqrt{\tfrac{1}{3}}\,|1,0\rangle |\tfrac{1}{2}-\tfrac{1}{2}\rangle - \sqrt{\tfrac{2}{3}}\,|1,-1\rangle |\tfrac{1}{2}+\tfrac{1}{2}\rangle$$

(2) In momentum rep, $\hat{x} \doteq_p i\hbar \frac{\partial}{\partial p}$

$$\left[-\frac{\hbar}{i}\frac{\partial}{\partial p}, p\right] = i\hbar$$

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2 \doteq_x -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{1}{2}m\omega^2 x^2$$

$$\doteq_p \frac{p^2}{2m} - \frac{1}{2}m\omega^2\frac{d^2}{dp^2}$$

which is the same with going from x to p

or $x \to p$, $\frac{1}{2m} \to \frac{1}{2}m\omega^2$

or $\frac{1}{m\omega} \to m\omega$

then $x_0 = \sqrt{\frac{\hbar}{m\omega}} \to \sqrt{\hbar m\omega} = p_0$

and so in momentum rep,

$$\psi_0(p) = \frac{1}{\pi^{1/4}\sqrt{p_0}} \exp\left(-\frac{1}{2}\left(\frac{p}{p_0}\right)^2\right)$$