Recitation #10
Quantum 521

1. In lecture 11 we will show how to decompose a direct product of two spin-1/2 for particle 1 and 2 ,

$$|1/2, m_1\rangle_1 \, |1/2, m_2\rangle_2$$

   into irreducible representations $j = 0, 1$ by applying the lowering operator $S_- = S_-^1 + S_-^2$ where the operator S_-^1 acts only on the 1-ket, and S_-^2 acts only on the 2-ket.

   For direct products of greater j, this becomes tedious. The coefficents in the decomposition are known as Clebsh-Gordon coefficients and can be found in tables. Use the table on the back of this page to get the decompositon for the two spin-1/2 for particles, and for the spin-1 combined with spin-1/2.

2. By comparing the hamiltonian of the simple harmonic oscillator in momentum-space to that of position space, find the momentum space wave function for the ground state. Recall that in position space the ground state is

$$\psi_0(x) = \frac{1}{\pi^{1/4}\sqrt{x_0}} \exp\left\{ -\frac{1}{2}\left(\frac{x}{x_0}\right)^2 \right\}$$

   where $x_0 = \sqrt{\frac{\hbar}{m\omega}}$

46. Clebsch-Gordan Coefficients, Spherical Harmonics, and *d* Functions

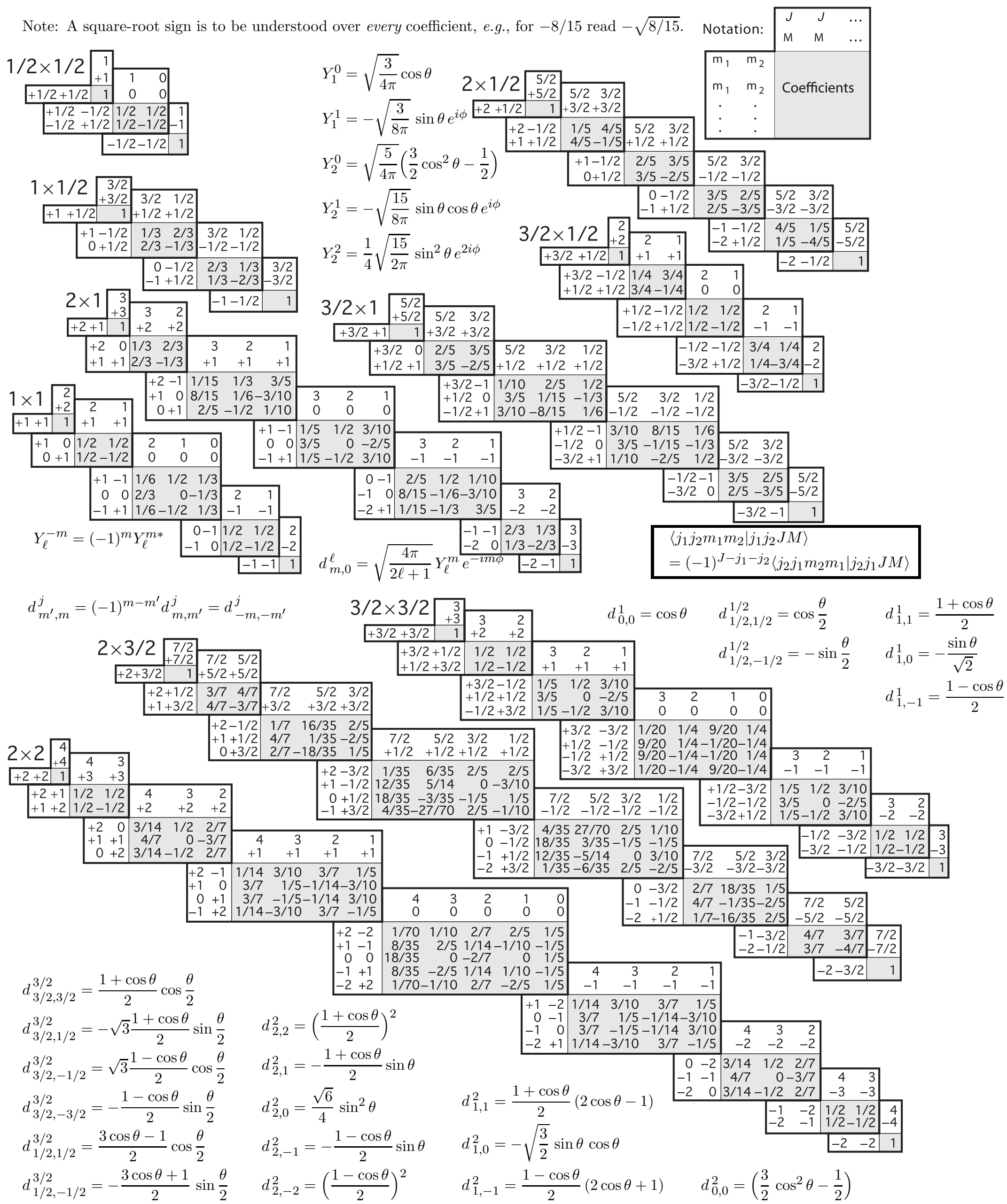

Figure 46.1: The sign convention is that of Wigner (*Group Theory*, Academic Press, New York, 1959), also used by Condon and Shortley (*The Theory of Atomic Spectra*, Cambridge Univ. Press, New York, 1953), Rose (*Elementary Theory of Angular Momentum*, Wiley, New York, 1957), and Cohen (*Tables of the Clebsch-Gordan Coefficients*, North American Rockwell Science Center, Thousand Oaks, Calif., 1974).