Phys 521 Fall 2023

Recitation 13 Solutions

$$\psi(\vec{r}) = (x + y + 3z) f(r)$$

(1)
$$Y_1^{\pm 1} = \mp \sqrt{\frac{3}{4\pi}} \frac{1}{r} \frac{x \pm iy}{\sqrt{2}}$$

$$Y_1^0 = \sqrt{\frac{3}{4\pi}} \frac{z}{r}$$

$$x = \sqrt{\frac{4\pi}{3}}\, r\, \frac{Y_1^{-1} - Y_1^1}{\sqrt{2}}$$

$$y = \sqrt{\frac{4\pi}{3}}\, r\, \frac{Y_1^{-1} + Y_1^1}{\sqrt{2}}$$

$$z = \sqrt{\frac{4\pi}{3}}\, r\, Y_1^0$$

$$\psi = \sqrt{\frac{4\pi}{3}}\, r \left[Y_1^1 \left(\frac{-1+i}{\sqrt{2}} \right) + Y_1^{-1} \left(\frac{1+i}{\sqrt{2}} \right) + 3 Y_1^0 \right]$$

$$\boxed{l = 1}$$

(2) Amplitudes $A_\pm = C \frac{i \pm 1}{\sqrt{2}}$; $A_0 = C\,3$

$$|A_\pm|^2 = C^2 \quad |A_0|^2 = 9C^2$$

normalizing $1 = C^2(1 + 1 + 9) \Rightarrow C^2 = \frac{1}{11}$

Probabilities $P_\pm = \frac{1}{11}$ $\quad P_0 = \frac{9}{11}$

Schrodinger

$$-\frac{\hbar^2}{2m}\left(\frac{1}{r}\frac{\partial}{\partial r} r\right)^2 \psi + \frac{L^2}{2mr^2}\psi + V\psi = E\psi$$

with $\psi = r f(r)\, g_1(\theta, \phi)$

$$-\frac{\hbar^2}{2m}\left(\frac{1}{r}\frac{\partial}{\partial r} r\right)^2 (rf) + \frac{\hbar^2}{mr^2}(rf) + V(rf) = Erf$$

cancel

$$-\frac{\hbar^2}{2m}\left(2f + 2rf' + f''\right) - \frac{\hbar^2 f}{m} + Vr^2 f = Er^2 f$$

$$V(r) = E + \frac{\hbar^2}{m}\left[\frac{f'}{rf} - \frac{1}{2}\frac{f''}{r^2 f}\right]$$