Phys 521 Fall 2023

Recitation #14 Solutions

① $|z\rangle = e^{-|z|^2/2} e^{za^+} |0\rangle$

from $e^A e^B e^{-C/2} = e^{A+B}$ $\quad C = [A,B]$

$[a^+, a] = -1$

with $A = za^+$, $B = -z^* a$, $C = -|z|^2 [a^+, a] = +|z|^2$

$$e^{za^+} e^{-z^* a} e^{-|z|^2/2} = \exp(za^+ - z^* a)$$

so $e^{-|z|^2/2} e^{za^+} = \exp(za^+ - z^* a) e^{z^* a}$

$$|z\rangle = e^{-|z|^2/2} e^{za^+} |0\rangle = \exp(za^+ - z^* a) e^{z^* a} |0\rangle$$

expanding $e^{z^* a} |0\rangle = (1 + z^* a + \cdots)|0\rangle = 0$

so $|z\rangle = \exp(za^+ - z^* a)|0\rangle$

(2) rewrite $\exp(za^+ - z^* a)|0\rangle$
in terms of $z = \frac{y_0 + i q_0}{\sqrt{2}}$

$$za^+ = \tfrac{1}{2}(y_0 + i q_0)(\hat{y} - i\hat{q})$$
$$= \tfrac{1}{2}(y_0\hat{y} + i q_0\hat{y} - i y_0\hat{q} + q_0 y_0)$$

$$(za^+ - z^* a) = i(q_0\hat{y} - y_0\hat{q})$$

then since $[iq_0\hat{y}, -iy_0\hat{q}] = -q_0 y_0[\hat{y},\hat{q}] = -iq_0 y_0$

$$\exp(iq_0\hat{y} - y_0\hat{q}) = e^{iq_0 y_0/2}\,\underbrace{e^{iq_0\hat{y}}}_{\hat{T}(q_0)}\,\underbrace{e^{-iy_0\hat{q}}}_{\hat{W}(y_0)}$$

spatial translation by q_0
momentum translation by y_0