Phys 521

Recitation #2 Solutions

1) Eigenvectors of hermitian operator A,

$$\hat{A}\,|a_i\rangle = a_i\,|a_i\rangle$$; labels different eigenvalues

operate with bra $\langle a_j|$

$$\langle a_j|\hat{A}|a_i\rangle = a_i\,\langle a_j|a_i\rangle \qquad *$$

exchange $i \rightarrow j$

$$\langle a_i|\hat{A}|a_j\rangle = a_j\,\langle a_i|a_j\rangle$$

take complex conjugate using $A = A^+$

$$\left(\langle a_i|\hat{A}|a_j\rangle\right)^* = \langle a_j|A^+|a_i\rangle = \langle a_j|A|a_i\rangle$$

$$= a_j^*\,\langle a_j|a_i\rangle = a_j\,\langle a_j|a_i\rangle$$

a_i are real

subtract * equation

$$0 = (a_j - a_i)\,\langle a_j|a_i\rangle$$

giving $\langle a_j|a_i\rangle = \delta_{ij}$

2) $[x, p] = i\hbar$

$$F(p) = \sum_{n=0}^{\infty} \frac{1}{n!} \frac{d^n F}{dp^n}\Big|_{p=0} p^n$$

$$[x, p^2] = xp^2 - p^2 x = (px + i\hbar)p - p^2 x$$

$$= i\hbar p + p(px + i\hbar) - p^2 x = 2i\hbar p$$

by induction, $[x, p^n] = n i\hbar p^{n-1}$

$$[x, F(p)] = \sum_{n=0}^{\infty} \frac{1}{n!} \frac{d^n F}{dp^n}\Big| [x, p^n]$$

$$= i\hbar \sum_{n=0}^{\infty} \frac{1}{n!} \frac{d^n F}{dp^n}\Big| n p^{n-1}$$

$$= i\hbar \frac{dF}{dp}$$

3) $\hat{x}$ does not depend explicitly on time so

$$\frac{d}{dt}\langle x \rangle = \frac{i}{\hbar} \langle [\hat{H}, x] \rangle$$

$$\hat{H} = \frac{1}{2}\frac{p^2}{m} \qquad [\hat{H}, \hat{x}] = -i\hbar \frac{p}{m}$$

$$\frac{d}{dt}\langle x \rangle = \frac{1}{m}\langle p \rangle$$