Recitation #3
Quantum 521

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1. Prove that if the Hamiltonian is time dependent but $\left[\hat{H}(t_2), \hat{H}(t_1)\right] = 0$

$$\exp\left(\frac{-\mathrm{i}}{\hbar}\int_0^{\mathrm{t}} \hat{\mathrm{H}}(\mathrm{t}')\mathrm{dt}'\right)$$

   Hint: for infinitessimal Δt

$$\hat{\mathrm{U}}\left(\mathrm{t}+\Delta\mathrm{t},\mathrm{t}\right) = \hat{\mathrm{I}} - \mathrm{i}\Delta\mathrm{t}\hat{\mathrm{H}}(\mathrm{t})/\hbar = \exp\left(-\mathrm{i}\Delta\mathrm{t}\mathrm{H}\hat{(}\mathrm{t})/\hbar\right)$$

2. Shankar postulates that the momentum operator is

$$\langle x|\,\hat{p}\,|x'\rangle = -i\hbar\delta'\left(x - x'\right)$$

   Use this to find a first order differenial equation for the plane wave state $\langle x|p\rangle = \phi_p(x)$. Integrate to find $\phi_p(x)$ up to a normalization constant. Determine the normalization constant from $\langle p|p'\rangle = \delta\left(p - p'\right)$