Phys 521 Fall 2023

Recitation #4 Solution

(1) Skew hermitian $A^+ = -A$

$$A|a_i\rangle = a_i|a_i\rangle$$
$$\langle a_j|A|a_i\rangle = a_i\delta_{ij}$$

$$\langle a_j|A^+ = a_j^* \langle a_j|$$

$$\langle a_j|A^+|a_i\rangle = -\langle a_j|A|a_i\rangle = a_j^*\delta_{ij} = a_i^*\delta_{ij}$$

then subtracting, $0 = (a_i + a_j^*)\delta_{ij}$

only possible value for a_i real is $a_i = 0$
At most 1 real eigenvalue (0) possibly degenerate.

(2) $\hat{A}'|\psi\rangle = c\hat{B}'|\psi\rangle \quad \{A', B'\} = 0$ So

$$\langle A'^2\rangle\langle B'^2\rangle \geq \frac{1}{4}|\langle[A', B']\rangle|^2$$

$$\langle A'^2\rangle = |c|^2\langle B'^2\rangle$$

$$\langle[A', B']\rangle = \langle\psi|(A'B' - B'A')|\psi\rangle =$$
$$(c^* - c)\langle\psi|B'^2|\psi\rangle$$

For equality to hold,

$$|c|^2 = \frac{1}{4}|(c^* - c)|^2 \quad \text{true if } c = ib$$
b real

For Gaussian wave function:

$$\hat{P}_x \psi(x) = \frac{\hbar}{i}\frac{d}{dx}\, \alpha \exp\left(ikx - \frac{x^2}{4\sigma^2}\right)$$

$$= \frac{\hbar}{i}\left(ik - \frac{x}{2\sigma^2}\right)\psi$$

$$\langle P_x \rangle = \hbar k$$

$$\left(\hat{P}_x - \hbar k\right)\psi = \frac{i\hbar x}{2\sigma^2}\psi$$

$$\langle x \rangle = |\alpha|^2 \int_{-\infty}^{\infty} e^{-x^2/2\sigma^2}\, x\, dx = 0$$

$$\hat{x}' = \hat{x}$$

so $$\hat{P}_x' \psi = \frac{i\hbar}{2\sigma^2}\hat{x}'\psi$$

constant is pure imaginary $\frac{i\hbar}{2\sigma^2}$