

Recitation #6 Solutions

① a) Particle in a box wave functions on interval $0 < x < a$:

$$\langle x | n \rangle = \varphi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right)$$

$$\psi(x) = \frac{3}{4} \varphi_1 + \frac{4i}{5} \varphi_3$$

$$P_1 = 9/25, \quad P_2 = 0, \quad P_3 = 16/25$$

$$(b) \quad \hat{H} \varphi_n = -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} \varphi_n = \frac{\hbar^2}{2m} \left(\frac{n\pi}{a}\right)^2 \varphi_n$$

$$(c) \quad \langle \hat{H} \rangle = \left[\frac{9}{25} \left(\frac{\pi^2}{a^2}\right) + \frac{16}{25} \left(\frac{3\pi}{a}\right)^2 \right] \frac{\hbar^2}{2m}$$

$$= E_1 \left(\frac{9}{25} + 9 \left(\frac{16}{25}\right) \right) = 6E_1 \quad E_1 = \frac{\hbar^2}{2m} \left(\frac{\pi}{a}\right)^2$$

$$(2) \quad \psi(x, 0) = \frac{1}{\sqrt{2}} (\varphi_1 + \varphi_2)$$

$$\psi(x, t) = \frac{1}{\sqrt{2}} \left(e^{-iE_1 t/\hbar} \varphi_1 + e^{-i4E_1 t/\hbar} \varphi_2 \right)$$

$$= \frac{1}{\sqrt{2}} e^{-iE_1 t/\hbar} \left(\varphi_1 + e^{-i\omega t} \varphi_2 \right) \quad \omega = \frac{3E_1}{\hbar}$$

$$\langle x \rangle = \frac{1}{2} \left[\langle 1|x|1 \rangle + \langle 2|x|2 \rangle + e^{+i\omega t} \langle 2|x|1 \rangle + e^{-i\omega t} \langle 1|x|2 \rangle \right]$$

$$= \frac{1}{2} \left[\langle 1|x|1 \rangle + \langle 2|x|2 \rangle + 2 \cos \omega t \langle 1|x|2 \rangle \right]$$

by symmetry $\langle 1|x|1 \rangle = \langle 2|x|2 \rangle = \frac{a}{2}$

$$\langle 2|x|1 \rangle = \langle 1|x|2 \rangle = \frac{2}{a} \int_0^a x \sin \frac{\pi x}{a} \sin \left(\frac{2\pi x}{a} \right) dx$$

$$\theta = \frac{\pi x}{a}$$

$$= \frac{2}{a} \left(\frac{a}{\pi} \right)^2 \int_0^{\pi} \theta \sin \theta \sin 2\theta d\theta$$

$$= \frac{2a}{\pi^2} \left(-\frac{8}{9} \right)$$

$$\langle x \rangle = \frac{1}{2} \left[\frac{a}{2} + \frac{a}{2} - \frac{4a}{\pi^2} \left(\frac{8}{9} \right) \cos(\omega t) \right]$$

$$= a \left[\frac{1}{2} - \frac{16}{9\pi^2} \cos(\omega t) \right]$$