Phys 521 Fall 2023

Recitation #7

(1) $B = \dfrac{k - iq}{k + iq}$ $\quad |B| = 1$

$$= \frac{k^2 - 2ikq - q^2}{k^2 + q^2} = e^{i\phi} \quad \tan\phi = \frac{-2kq}{k^2 - q^2}$$

$$\psi_- = e^{ikx} + e^{i\phi} e^{-ikx}$$

$$\psi_+ = C e^{-qx} = (1 + e^{i\phi}) e^{-qx} = e^{i\phi/2}(e^{-i\phi/2} + e^{i\phi/2}) e^{-qx}$$

$1 + B = C$

$$= 2 e^{i\phi/2} \cos\frac{\phi}{2} e^{-qx}$$

$$\lambda = \frac{1}{2q} \qquad E = V_0/2, \ \hbar q = \sqrt{mV_0}$$

$V_0 = 16\,\text{eV}$ $\qquad m = \frac{1}{2} \times 10^6\,\text{eV}$

$$\lambda^{-1} = 2 \frac{\sqrt{mV_0}}{\hbar c}$$

$$\lambda^{-1} = \left(\frac{2}{200\,\text{eV}\cdot\text{nm}}\right) \sqrt{10^6\,\text{eV}\ \tfrac{1}{2}(8\,\text{eV})} = 20\,\text{nm}^{-1}$$

$$\lambda = \frac{1}{20}\,\text{nm}$$

$$B \xrightarrow[k \ll q]{} = -1 \qquad \phi = \pi$$

$$\psi_+ = 2\sin kx \ ; \quad \psi_- = 0$$

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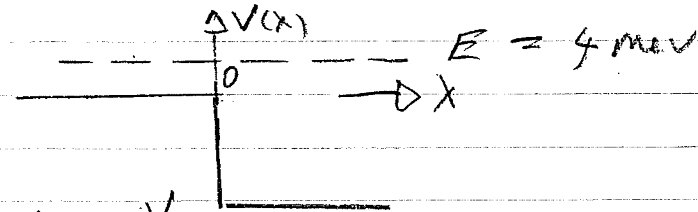
(2)



$-12\,\text{mev} = -V_0$

$$\psi_- = e^{ikx} + Be^{-ikx} \qquad \psi_+ = Ce^{ik'x}$$

$$\hbar k = \sqrt{2mE} \qquad \hbar k' = \sqrt{2m(E+V_0)}$$

$$\frac{k'}{k} = \sqrt{\frac{4+12}{4}} = 2$$

$$T = \frac{k'}{k}|C|^2$$

$$1 + B = C \qquad B = C - 1$$

$$ik(1-B) = ik'C \qquad (1-B) = \frac{k'}{k}C$$

$$2 - C = \frac{k'}{k}C \Rightarrow C = \left(\frac{2}{\frac{k'}{k}+1}\right)$$

$$T = 2\left|\frac{2}{2+1}\right|^2 = \frac{8}{9}$$

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(3)

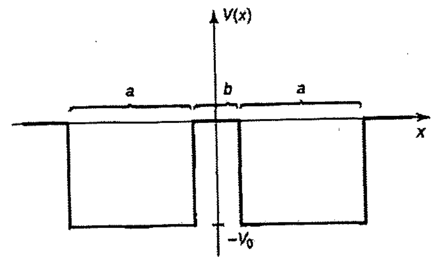
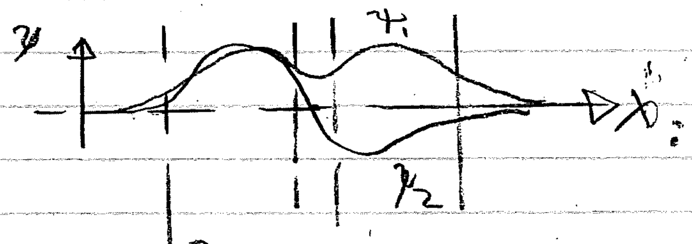


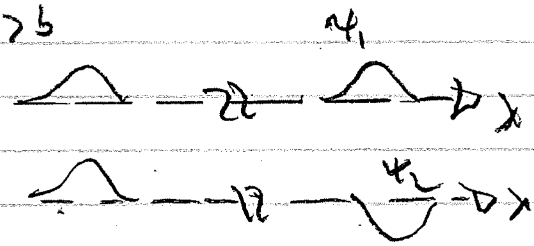
Figure 1: square double well potential (from Griffiths)



$$\left\langle \frac{d^2\psi_2}{dx^2} \right\rangle > \left\langle \frac{d^2\psi_1}{dx^2} \right\rangle$$

more curvature less curvature

in limit $a >> b$



degenerate states

$E_2 = E_1$

recall NH_3 descriptive Hamiltonian in $|1\rangle, |2\rangle$ basis:

$$[H]_{12} = \begin{pmatrix} E_0 & -A \\ -A & E_0 \end{pmatrix}$$

minus A gives symmetric ground state

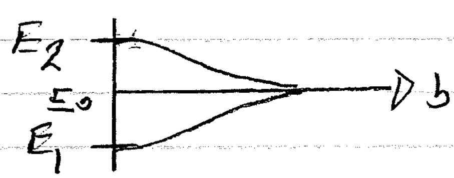
Eigenvalues $E_\pm = E_0 \pm A$ eigenstates $|I\rangle = \frac{1}{\sqrt{2}}(|1\rangle + |2\rangle)$

$E_- = E_0 - A$

In limit $A \to 0$, $E_\pm = E_0$

$|II\rangle = \frac{1}{\sqrt{2}}(|1\rangle - |2\rangle)$

$E_+ = E_0 + A$



double square well energy eigenvalues versus b