## Recitation #7 Quantum 521

1. Consider scattering off of a step-up potential with  $E < V_0$ . We found the reflected wave amplitude to be

$$B = \frac{k - iq}{k + iq}$$

where  $\hbar k = \sqrt{2mE}$  and  $\hbar q = \sqrt{2m(V_0 - E)}$ 

a) Show that  $B = e^{i\phi}$ , a pure phase. Write the wave functions before  $\psi_{-}$  and after  $\psi_{+}$  the step (at x = 0) in terms of  $\phi$ .

b) Find the mean (1/e) penetration depth (in nm) into the barrier for an electron with  $E = V_0/2$ ,  $V_0 = 8$  eV. (Numbers are simple so you do not need a calculator).

- c) Find  $\phi$  and  $\psi_{\pm}$  in the limit  $E \ll V_0$ . (Be careful evaluating the tan<sup>-1</sup>)
- 2. Neutron absorption can be modeled as a collision with a step down potential.

$$V(x) = 0 \qquad x < 0$$
  
$$V(x) = -V_0 \qquad x > 0$$

Calculate the neutron absorption probability as the probability for transmission through the barrier. Take E = 4 MeV and  $V_0 = 12$  MeV.

3. Consider the square double well potential shown in the figure. Sketch the ground state and first excited state wave functions for the two cases  $b \approx a$  and  $b \gg a$ . Based on the curvature of the wave function, argue that the symmetric state is the ground state.

$$\mathrm{KE} = \frac{-\hbar^2}{2\mathrm{m}} \langle \frac{\mathrm{d}^2}{\mathrm{d}\mathrm{x}^2} \rangle$$

Sketch the corresponding energies  $E_1, E_2$  as functions of b on the same graph.



Figure 1: square double well potential (from Griffiths)