Phys 521 Fall 2023

Recitation #8 Solutions

(a) Difference in action along 2 paths

$$S = \int dt \left(\frac{1}{2} m \dot{x}^2 - mgz \right)$$

x is distance traveled, neutron has some speed along each segment, so taking $z = 0$ along $\overline{AC}$

$$S_{ABD} = S_0 - mg l_1 l_2 \sin\delta \, T$$

$$S_{ACD} = S_0$$

$$\frac{\Delta S}{\hbar} = -\frac{mg l_1 l_2 \sin\delta \, T}{\hbar}$$

with $p = \frac{\hbar}{\lambda}$ $\quad T = \frac{l_1}{v} = \frac{l_1 m \lambda}{\hbar}$

$$\Delta\phi = \phi_{ABD} - \phi_{ACD} = -\frac{m^2 g l_1 l_2 \lambda \sin\delta}{\hbar^2}$$

$$\psi_1 + \psi_2 = \psi_0 \left(1 + e^{i\Delta\phi}\right)$$

$$P = |\psi_1 + \psi_2|^2 = \frac{1}{2}\left(1 + \cos\Delta\phi\right)$$

② propagator can be written as:

$$\langle x_1, t \mid x_0, 0\rangle = \int dp \, \langle x_1 | p\rangle \langle p | x_0\rangle e^{-iEt/\hbar}$$

for free particle, $E = \frac{p^2}{2m}$, p plane wave basis state

$$\langle x | p\rangle = \frac{1}{\sqrt{2\pi\hbar}} e^{ipx/\hbar}$$

$$\langle x_1, t \mid x_0, 0\rangle = \int \frac{dp}{2\pi\hbar} e^{ip(x_1 - x_0)/\hbar} e^{-i\frac{p^2 t}{2m\hbar}}$$

arg of exponent is

$$\frac{i}{\hbar}\left[p(x_1 - x_0) - \frac{p^2 t}{2m}\right] \qquad a = \frac{it}{2mi\hbar}, \; b = \frac{i}{\hbar}(x_1 - x_0)$$

$$\langle x_1, t \mid x_0, 0\rangle = \sqrt{\frac{\pi 2m\hbar}{it}}\left(\frac{1}{2\pi\hbar}\right) \exp\left[\frac{im(x_1 - x_0)^2}{2\hbar t}\right]$$

$$\sqrt{\frac{m}{2\pi i\hbar t}}$$