

Recitation #9 Solutions

$$\textcircled{1} \quad |\psi\rangle = A|0\rangle + B|1\rangle \quad \text{S.H.O. state}$$

since $|A|^2 + |B|^2 = 1$, can write a

$$|\psi\rangle = \cos\theta |0\rangle + \sin\theta |1\rangle$$

$$\text{with } \hat{x} = \frac{1}{\sqrt{2}}(a + a^\dagger)$$

$$\langle \hat{x} \rangle = \frac{1}{\sqrt{2}} (\cos\theta \sin\theta + \sin\theta \cos\theta)$$

$$= \frac{1}{\sqrt{2}} \sin(2\theta) \quad \text{max } 2\theta = \frac{\pi}{2}$$

$$\theta = \pi/4$$

$$\text{thus } |\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$\textcircled{2} \quad |\psi(t)\rangle = \frac{1}{\sqrt{2}} e^{-i\omega_0 t} (|0\rangle + e^{-i\omega_0 t} |1\rangle)$$

$$\langle \psi(t) | \hat{x} | \psi(t) \rangle =$$

$$\frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} (\langle 0| + e^{i\omega_0 t} \langle 1|) (a + a^\dagger) (|0\rangle + e^{-i\omega_0 t} |1\rangle)$$

$$= \frac{1}{2\sqrt{2}} (e^{-i\omega_0 t} + e^{i\omega_0 t}) = \frac{1}{\sqrt{2}} \cos(\omega_0 t)$$